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Izboljšana razpoznava dušenja z uporabo zvezne valčne transformacije

Enhanced identification of damping using continuous wavelet transform

Janko Slavič - Miha Boltežar

Pri določitvi parametrov dinamičnih sistemov z več prostostnimi stopnjami se izkaže, da je razpoznava dušilnih parametrov težja kakor razpoznava masnih in togostnih parametrov. V tem prispevku se osredotočimo na razpoznavanje dušenja z uporabo hitro razvijajoče se zvezne valčne transformacije (ZVT). Gaborjeva ZVT je temelj za tri izboljšane postopke identifikacije: metodo zmanjšanega časovnega raztrosa valčne funkcije, metodo enakovredne površine okna in metodo zrcaljenja okna. Predstavljene metode so posebej primerne za kratke in močno dušene sisteme, pri katerih je vpliv roba moteč. Predstavljeni postopki so prikazani na dejanskem eksperimentu, pri katerem smo izmerili razmernik dušenja prvih šestih lastnih frekvenc.

(Ključne besede: sistemi dinamični, ugotavljanje dušenja, transformacija valčkov, postopki ugotavljanja)

A continuous wavelet transform (CWT) based on the Gabor wavelet function is used to identify the damping of a multi-degree-of-freedom system. The basic procedures are already known, especially the identification with a Morlet CWT. This paper enhances the already known procedures in the sense of reducing the edge-effect. This makes it possible to identify the damping of high-frequency modes. Three methods are shown: the reduced time spread of the wavelet function method, the equivalent window area method and the reflected window method. The procedures are demonstrated on signals acquired from the lateral vibration of a uniform beam. We were able to identify the damping up to the sixth natural frequency.

(Keywords: dynamical systems, damping identification, wavelet transformation, identification methods)

0 UVOD

Raztros vibracijske energije imenujemo dušenje. Pretežni del vibracijske energije se porazgubi znotraj sistema, predvsem v obliki toplote; preostali del energije se porazgubi v obliki akustičnega sevanja, prenosa energije na druge sisteme itn.

Delo, ki ga naredi nihajoča sila dušenja pri enem krogu, imenujemo raztros energije enega kroga. Poskusi so pokazali, da je raztros energije kroga za večino konstrukcijskih materialov — kakor sta jeklo in aluminij — v širokem frekvenčnem spektru neodvisna od frekvence nihanja in da je raztros proporcionalen kvadratu amplitude nihanja. Tako notranje dušenje imenujemo strukturno dušenje [1]. V dinamiki nihanj najpogosteje uporabljen model (zaradi enostavnosti) *viskoznega dušenja* ne ustreza tem kriterijem, saj je sila dušenja sorazmerna hitrosti nihanja in posledično je raztros energije dušenja odvisen od frekvence nihanja. Da se torej zadovolji kriterijem strukturnega dušenja, se vpelje

0 INTRODUCTION

The damping of dynamic systems is the dissipation of vibration energy. Usually, a considerable amount of this energy dissipates inside the system, mostly in the form of heat, while the rest dissipates outside the system in the form of acoustic radiation, transmission to other dynamic systems, etc.

Compared to an estimation of the stiffness and the mass properties of multi-degree-of-freedom (MDOF) systems, an estimation of the damping parameters is more difficult. The factors affecting damping mechanisms include friction on the atomic/molecular level, dry friction, viscous friction in fluids, etc., and so it is often difficult to describe in detail the real physical background using mathematical means. As a consequence of this, a number of simplified models were developed. Of these models, the model of *viscous damping* is the most widely used, it assumes that the damping force is proportional to enakovredno viskozno dušenje. Ustrezno viskozno dušenje se dobi tako, da se (ob predpostavki harmoničnega nihanja) teoretično raztros energije kroga viskoznega dušenja izenači z dejanskim raztrosom energije enega kroga neviskoznega nihanja.

Kakorkoli, dušenja ne moremo izmeriti neposredno, ampak ga dobimo z vrednotenjem odziva dinamičnega signala. To lahko naredimo v časovnem ali v frekvenčnem območju. Logaritmični dekrement je najpreprostejši postopek in se izvaja v časovnem območju; uporaben je predvsem za sisteme z eno prostostno stopnjo (EPS). Bolj zapleteni metodi v časovnem območju, ki sta uporabni tudi za sisteme z več prostostnimi stopnjami (VPS), sta metoda najmanjših kvadratov po Smith-u in metoda najmanjših kvadratov s kompleksnim eksponentom — obe podrobneje opisani v [2]. Pogosto uporabljena časovna metoda je tudi Hilbertova transformacija [3], ki spremeni realno funkcijo v kompleksno.

Med metodami v frekvenčnem območju je znana t.i. metoda 3dB [4], ki temelji na pasovni širini resonančnega vrha. Za izboljšanje rezultatov te metode so razvite tudi bolj izpopolnjene metode in so navedene v [5].

Sicer pa Staszewski [5] navaja, da lahko dušenje ocenimo tudi z uporabo časovno-frekvenčne Wigner-Ville-ove porazdelitve.

Drugačen način predstavlja uporaba valčne transformacije; primer takega načina je predstavil Staszewski [5]. Gre za uporabo časovno in frekvenčno omejenih funkcij, ki jih premikamo po časovni osi in stopnjevanje po frekvenčni osi. S superpozicijo takih funkcij lahko popišemo celoten signal. V tem primeru ne govorimo o časovno-frekvenčnem območju, temveč o območju čas-stopnjevanje. Tako pridemo do valčne transformacije, ki jo je leta 1980 predstavil Morlet [6] in jo uporabil za seizmološke raziskave. Matematični formalizem za zvezno valčno transformacijo sta razvila skupaj Grossman in Morlet [7]. Od takrat naprej se je zvezna valčna transformacija (ZVT) hitro razvijala in se danes uporablja na zelo širokem področju: od opazovanja neustaljenih in nelinearnih pojavov, prek identifikacije napak in uporabe v kombinaciji z nevronskimi mrežami, do reševanja diferencialnih enačb z uporabo valčne transformacije. Podrobnejši pregled uporabe je predstavljen v [8].

Prednost uporabe valčne transformacije je prav v njeni časovni in frekvenčni omejenosti, ki omogoča preprosto analiziranje sistemov z VPS; lokalna omejenost valčne transformacije namreč omogoča, da analiziramo posamezne lastne frekvence neodvisno od drugih. Prednost valčne transformacije je tudi njena odpornost proti šumu v signalu [9].

V poglavju 1 bodo predstavljene osnove valčne transformacije in uporabe valčne transformacije za razpoznavo dušenja. V poglavju 2 bo na kratko predstavljen vpliv roba. V poglavju 4 bomo predstavili tri metode razpoznave določitve

the velocity of oscillation; and so it follows that the work done by one oscillation cycle depends on the frequency of the oscillation. Another often-used model is the model of *structural damping*, where the work done in one cycle is independent of the oscillation frequency and where the dissipation of the vibrational energy is proportional to the square of the amplitude [1]. To overcome the shortcomings of the different models the model of equivalent viscous damping is used. In this paper the damping is discussed in terms of the damping ratio, i.e. the fraction of critical damping.

Because the damping is hidden in the response of a dynamic system it cannot be measured directly. A number of damping measures and criteria in the time and frequency domains are used to characterize structural damping. Logarithmic decrement – probably the simplest method in the time domain – is useful for single-degree-of-freedom (SDOF) systems. Often-used time-domain methods, suitable for identifying the damping of multi-degree-of-freedom (MDOF) systems are: the Smith least squares [2], the least-squares complex exponential [2] and the Hilbert transform [3].

The simplest frequency domain method is the 3dB method [4]. The damping is estimated from the frequency-response width; however, because of the low accuracy some enhancements have been developed [5].

The third way is the time-frequency domain. For example the Wigner-Ville distribution [5].

A different method in the time-scale domain is the use of the continuous wavelet transform (CWT) [5]. The CWT is based on functions limited in time and frequency that are translated on the time axis and scaled on the frequency axis. With a superposition of such functions the whole signal can be described. The CWT was introduced in 1980 by Morlet [6]. The mathematical definitions were developed by Grossman and Morlet [7]. Today, the CWT is rapidly developing and has spread to a wide range of applications [8]: the observation of nonstationary and nonlinear processes, the identification of faults in complex systems, applications in neural networks and differential equations, etc.

The major advantage of the CWT is the locality in time and frequency that can be used to individually analyze each degree of freedom of a MDOF system. The CWT was shown to be very efficient with noisy signals [9].

In the second section some basics of the CWT, and the identification of damping with the help of the CWT, will be presented. The third section gives a short explanation of the edge-effect. In the fourth section three new methods for the identification of damping with the help of the CWT are given: the reduced time spread of the wavelet function method, dušenja: metoda zmanjšanega časovnega raztrosa valčne funkcije, metodo zrcaljenja okna in metodo enakovredne površine okna. V poglavju 4 so predstavljeni rezultati preskusa ter v 5 sklepi.

the reflected window method and the equivalent window area method. The fifth section presents an experiment and the last section presents the conclusions.

1 RAZPOZNAVA RAZTROSA ENERGIJE Z VALČNO TRANSFORMACIJO

1 IDENTIFICATION OF THE DISSIPATION ENERGY BY CWT

1.1 Osnove valčne transformacije

1.1 Basics of the wavelet transform

Tukaj navajamo zgolj nekatere osnovne definicije zvezne valčne transformacije, za podrobnejšo predstavitev se naj bralec obrne na drugo literaturo; npr.: [8], [10] in [11].

ZVT funkcije $x(t)\in L^2(\mathbf{R})$ je določena kot:

In this subsection only a few basic definitions are presented, for an exhaustive study the reader should refer to other literature, e.g. [8], [10] and [11].

The CWT of the signal $x(t)\in L^2(\mathbf{R})$ is defined as:

$$Wx(u,s)=\int_{-\infty}^{+\infty} x(t)\psi_{u,s}^{*}(t)\,\mathrm{d}t \quad (1),$$

kjer sta u in s parametra premika in stopnjevanja [12]. $\psi^*(t)$ je kompleksno konjugirana osnovna valčna funkcija (VF) $\psi(t)\in L^2(\mathbf{R})$.

Valčna funkcija je normirana (njena norma je enaka 1). Povprečna vrednost VF je 0:

where u and s are the translation and scale/dilation parameters, respectively [12], and $\psi^*(t)$ is the complex conjugate of the basic wavelet function $\psi(t)\in L^2(\mathbf{R})$.

The wavelet function is a normalized function (i.e. the norm is equal to 1) with an average value of zero:

$$\|\psi(t)\|^2=\int_{-\infty}^{+\infty}|\psi(t)|\,\mathrm{d}t=1 \quad (2)$$

$$\overline{\psi}(t)=\int_{-\infty}^{+\infty}\psi(t)\,\mathrm{d}t=0 \quad (3).$$

Preglednica 1 prikazuje normo in povprečno vrednost Morletove in Gaborjeve VF [13]. Medtem ko je v obeh primerih povprečna vrednost ob primerni izbiri parametrov σ in η zelo blizu nič, je norma Gaborjeve VF enaka 1, norma Morletove VF pa π. Morletovo VF lahko normiramo z množenjem z $1/\sqrt[4]{\pi}$. V tem primeru dobimo Gaborjevo VF s parametrom $\sigma = 1$. Parameter σ omogoča, da prilagodimo časovni in frekvenčni raztros, to lastnost bomo pozneje podrobneje predstavili.

Premaknjena in stopnjevana VF je definirana kot:

Table 1 shows the norm and the mean values of the Morlet and Gabor wavelet functions [13]. In both cases the selection of suitable parameters σ and η makes the mean value very close to zero. While the norm of the Gabor wavelet is equal to 1, the norm of the Morlet wavelet is π. However, we can normalize the Morlet wavelet function by multiplying it by $1/\sqrt[4]{\pi}$. The normalized Morlet wavelet function is identical to the Gabor wavelet function with the parameter $\sigma = 1$. The additional parameter σ of the Gabor wavelet function gives us the possibility to adapt the time and frequency spread and will be discussed later in this paper.

The translated and dilated wavelet function is defined as:

$$\psi_{u,s}(t)=\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \quad (4)$$

Gaborjeva VF kot:

The Gabor wavelet function is defined as:

$$\psi_{Gabor}(t)=\underbrace{\frac{1}{(\sigma^2\pi)^{1/4}}e^{-\frac{t^2}{2\sigma^2}}}_{\text{Gaussovo okno}}\cdot e^{i\eta t} \quad (5),$$

Preglednica 1. *Norma in povprečna vrednost Morletove in Gaborjeve valčne funkcije*
Table 1. *The norm and the mean value of the Morlet and Gabor wavelet function*

Lastnost/Property	Gabor	Morlet
$\|\psi(t)\|^2$	1	1
$\overline{\psi}(t)$	$\sqrt[4]{4\pi\sigma^2}\,e^{-\frac{\eta^2\sigma^2}{2}}$	$e^{-\frac{\eta^2}{2}}\sqrt{2\pi}$

kjer parameter σ in začetna skala definirata časovni in frekvenčni raztros Gaborjeve VF [14]; η je parameter frekvenčne prilagoditve.

Zaradi lažje primerjave bomo v nadaljevanju uporabljali tudi izraz "normiran parameter σ" (σ_{1Hz}), ki pove, kakšen parameter σ bi uporabili, če bi analizirali signal s frekvenco 1 Hz. Dejansko uporabljen parameter σ dobimo z množenjem: $\sigma_{1Hz} \cdot \Delta t$, kjer je Δt časovna diskretizacija.

Povezava med stopnjevanjem Gaborjeve VF s in krožno frekvenco ω definira izraz:

where parameter σ and the initial scale define the time and frequency spreads of the Gabor wavelet function [14]; η is the parameter of frequency modulation.

In this study the term "normalized parameter σ" (σ_{1Hz}) is used, this is the σ that would be used in the case of a signal with a frequency of 1 Hz. The appropriate parameter σ for any other frequency is: $\sigma_{1Hz} \cdot \Delta t$, where Δt is the time step of the discretization.

The relation between the instantaneous scale s and the instantaneous angular velocity ω of the Gabor wavelet function is defined as:

$$\omega(s) = \frac{\eta}{s} \tag{6}$$

Tukaj lahko navedemo še zelo uporabno lastnost ZVT, namreč njeno linearnost:

A very useful property of the CWT is its linearity:

$$\left(W \sum_{i=1}^{N} \alpha_i \, x_i \right)(u,s) = \alpha_i \sum_{i=1}^{N} \left(W \, x_i \right)(u,s) \tag{7}$$

Linearnost omogoči, da analiziramo i-to komponento x_i večkomponentne funkcije $\sum_{i=1}^{N} \alpha_i x_i$ posebej, ločeno od drugih; α_i je stalnica.

which makes it possible to analyze each i-th component x_i of a multi-component signal $\sum_{i=1}^{N} \alpha_i x_i$, where α_i is a constant.

1.2 Razpoznava raztrosa energije z uporabo Gaborjeve zvezne valčne transformacije

1.2 Damping identification using a wavelet transform

Razpoznava temelji na predpostavki, da sistem niha s harmonsko funkcijo $x(t)$ (8) in da je taka funkcija asimptotična, to je, da se amplituda glede na fazo spreminja počasi [15]:

We assume that the signal $x(t)$ is sinusoidal (8) and asymptotic, i.e. the signal's amplitude varies slowly compared to the variation of its phase [15]:

$$x(t) = A(t) \cos \varphi(t) \tag{8}$$

ZVT tako definirane harmonske funkcije lahko poenostavimo ; Staszewski [5] in Ruzzene s sodelavci [16] sta uporabila Morletovo VF in sta ZVT poenostavila v izraz, ki ga je predstavil Delprat s sodelavci [17]. Poenostavitev ZVT za Gaborjevo VF smo že predstavili [9], tako bo tukaj izpisan samo izpeljan izraz:

In the case of such a signal its CWT can be approximated by a simple function. Staszewski [5] and Ruzzene et al. [16] used the Morlet wavelet function, and as a consequence they used the CWT approximation as defined by Delprat et al. [17]. In this paper the approximation of the CWT based on the Gabor wavelet function is used [9]:

$$Wx(u,s) = \frac{1}{2} A(u) \, \hat{\psi}_{Gabor_{u,s}}(\varphi'(u), \sigma, \eta) \, e^{i\varphi(u)} + Er\left(A'(t), \varphi''(u) \right) \tag{9}$$

kjer je Fourierjeva transformacija premaknjene in povečane Gaborjeve VF definirana kot:

where the Fourier transform of the translated-and-scaled Gabor wavelet function is defined as:

$$\hat{\psi}_{Gabor_{u,s}}(\omega, \sigma, \eta) = (4\pi\, \sigma^2 s^2)^{1/4} \, e^{-\frac{\left(\omega - \frac{\eta}{s}\right)^2 \sigma^2 s^2}{2}} \, e^{-i\omega u} \tag{10}$$

Napako poenostavitve $\mathrm{Er}(A'(t), \varphi''(u))$ lahko zanemarimo, če je prvi odvod faze (kotna hitrost) večji od pasovne širine VF $\Delta\omega$ [10, 9]:

The approximation error $\mathrm{Er}(A'(t), \varphi''(u))$ can be neglected if the derivative of the phase is greater than the bandwidth $\Delta\omega$ [10, 9]:

$$\varphi'(u) \geq \Delta\omega \tag{11}$$

Pasovna širina Gaborjeve VF $\Delta\omega$ je definirana [10] in [9]:

The bandwidth $\Delta\omega$ of the translated-and-scaled Gabor wavelet function is defined as [10]:

$$\Delta\omega(s)=\sqrt{\frac{-2\,z_1}{\sigma^2 s^2}} \tag{12}$$

Izraz je izpeljan relativno, glede na velikost Gaussovega okna na sredini (največja vrednost) in na robu. Parameter z_1 predstavlja mero, kako daleč od sredine okna gledamo. V tem prispevku je bila uporabljena vrednost z_1= -8 (takrat je vrednost na robu okna 0,034% vrednosti na sredini).

Da se dve sosednji harmonski funkciji *i* in *j* v ZVT ne motita, mora biti večja pasovna širina od obeh ($\Delta\omega(s_i)$ in $\Delta\omega(s_j)$) manjša, kakor je frekvenčna razlika obeh funkcij [10]:

where the parameter z_1 needs to be chosen; if we choose z_1= -8 (the value used in this study) then the value of the wavelet at the bandwidth is only 0.034% of the maximum value.

For the CWT of any two components *i* and *j* of a multi-component signal not to interfere, the maximum of the bandwidth ($\Delta\omega(s_i)$ and $\Delta\omega(s_j)$) should be smaller than the frequency difference of *i* and *j* [10]:

$$(\varphi'_i(u)-\varphi'_j(u))\geq \max\left\{\Delta\omega(s_i),\Delta\omega(s_j)\right\} \tag{13}$$

ZVT neke harmonske funkcije v prikazu čas-stopnjevanje se kaže kot zgostitev energije pri frekvenci te harmonske funkcije. To zgostitev energije imenujemo greben [8] in jo opišemo s krivuljami $s=s(u)$. Greben je funkcija premika (časa). Vrednosti ZVT, ki so vezane na greben, imenujemo ogrodje ZVT – $Wx(u,s(u))$.

V nadaljevanju se bomo osredotočili na odziv dušenega nihanja [1]:

The time-scale representation of the energy concentration of the CWT is called the ridge. Ridges are described with the use of curves $s=s(u)$. In other words, ridges represent the frequency content of the analyzing signal with a high density of energy, which is dependent on the time (translation u). The values of the CWT that are restricted to the ridge are called the skeleton of the CWT – $Wx(u,s(u))$.

From now on we focus on the free response of a damped signal [1]:

$$x(t)=A_0 e^{-\zeta\omega_0 t}\cos(\omega_0\sqrt{1-\zeta^2}\,t+\phi) \tag{14}$$

Z uporabo ZVT smo zmožni določiti samo dušeno lastno krožno frekvenco $\omega_d=\omega_0\sqrt{1-\zeta^2}$, ki se po navadi od lastne krožne frekvence ω_0 (ki jo potrebujemo za izračun razmernika dušenja) razlikuje zelo malo. V nadaljevanju bomo tako uporabili ω_d namesto ω_0, saj je razmernik dušenja ζ navadno zelo majhen (ζ<<1) in napaka, ki jo s tem naredimo, zanemarljiva. Taka poenostavitev je v dinamiki pogosta.

Od sedaj naprej je postopek do razmernika dušenja enak kakor v primeru Morletove VF [5]. Najprej izraz (10) vstavimo v izraz (9) in zanemarimo napako. Ker nas zanima ZVT na mestu grebena ($s=s(u)$), je krožna frekvenca Gabor VF ($\omega=\eta/s$) enaka krožni frekvenci dušenega nihanja (ω_d); posledično je člen $e^{-\frac{(\omega_d-\eta/s)^2\sigma^2 s_0^2}{2}}$ enak 1. Tako izpeljemo naslednji izraz:

With the CWT we are able to determine only the damped angular velocity $\omega_d=\omega_0\sqrt{1-\zeta^2}$, which differs slightly from the undamped natural angular velocity ω_0. In future derivations we are forced to use ω_d instead of ω_0; however, because the damping ratio ζ is usually small (ζ<<1) the error is insignificant. In dynamics this substitution is usual.

The procedure for damping-ratio extraction is now just the same as in the case of the Morlet wavelet [5]. First, equation (10) is inserted into equation (9), and the error part is neglected. Because we paid attention to the ridge, the angular velocity of the Gabor wavelet function ($\omega=\eta/s$) is equal to the angular velocity of the signal (ω_d), and as a consequence the term $e^{-\frac{(\omega_d-\eta/s)^2\sigma^2 s_0^2}{2}}$ is equal to 1. Finally, the following expression is derived:

$$\ln\left(\frac{2\,|Wx(u,s(u))|}{(4\pi\,\sigma^2 s(u)^2)^{1/4}}\right)\approx -\zeta\omega_d\,u+\ln A_0 \tag{15}$$

Ko imamo enkrat izračunano ZVT, so vsi členi enačbe (15), razen razmernika dušenja ζ in stalnice A_0, znani. Ker gre za linearno funkcijo, lahko iz strmine te funkcije ocenimo razmernik dušenja.

Nihanje lahko rekonstruiramo z uporabo naslednjih izrazov:

It is clear that this equation represents a linear function and from its slope the damping ratio ζ can be estimated.

To reconstruct the signal the following functions can be derived:

$$\varphi(u)=\arctan\frac{\mathrm{Im}(Wx(u,s(u)))}{\mathrm{Re}(Wx(u,s(u)))} \tag{16}$$

$$A(u)\approx\frac{2\,|Wx(u,s(u))|}{(4\pi\,\sigma^2 s(u)^2)^{1/4}} \tag{17}$$

Edina naloga pri zgornjem postopku je določitev grebena $s(u)$.

1.3 Določevanje grebena

Podrobnejše informacije o različnih metodah določevanja grebena so zbrane v literaturi ([5], [15] do [18]). Tukaj bomo uporabili metodo prečnega preseka, ki temelji na vnaprej znani dušeni lastni frekvenci ω_d. Greben je tako definiran z:

Now the only problem is the characterization of the ridge $s(u)$.

1.3 Ridge detection

Detailed explanations of the various methods for ridge extraction can be found elsewhere ([5], [15] to [18]). The cross-sections method is the simplest method and will be used in this paper. The ridge is defined by:

$$s(u)=\frac{\eta}{\omega_d} \quad (18).$$

2 VPLIV ROBA

V našem preteklem delu [9] smo podrobneje proučili vpliv roba. Tukaj bomo za definiranje področja, kjer je vpliv roba opazen, uporabili polmer zaupanja, ki ga definiramo kot [19]:

2 THE EDGE-EFFECT

In our previous work [9] we studied the edge-effect in detail. The range where the edge-effect cannot be neglected is defined by the radius of trust [19]:

$$R(k,s,\sigma)=k\,\sigma_{t_{u,s}}=k\,\frac{\sigma\,s}{2} \quad (19),$$

kjer k pomeni večkratnik časovnega raztrosa Gaborjeve valčne funkcije. Z večanjem parametra k se veča površina valčne funkcije znotraj signala. Enota polmera zaupanja R je s (sekunda) in pomeni razpon vpliva roba v času.

where k defines the multiple of the time spread. With a higher k the area of wavelet function inside the signal rises. The unit of the radius of trust R is s (second).

3 IZBOLJŠANA RAZPOZNAVA DUŠENJA

Zaradi napake roba smo prisiljeni v razpoznavo dušenja na tistem delu valčne transformacije, kjer je vpliv roba zanemarljiv ([9] in [5]).

Tukaj bomo predstavili tri nove metode, s katerimi bomo poskušali zmanjšati vpliv roba. Te tri metode so: zmanjšanega raztrosa valčne funkcije, metoda zrcaljenja okna in metoda enakovredne površine okna.

3 ENHANCED IDENTIFICATION OF DAMPING

Because of the edge-effect error of the CWT we cannot identify the damping on a whole time inteval ([9] and [5]).

We will show three new methods that reduce the edge-effect. These three methods are: the reduced-time spread of the wavelet function method (RTS), the reflected window method (RFW) and the equivalent window area (EWA) method.

3.1 Metoda zmanjšanega časovnega raztrosa valčne funkcije – ZČR

Glede na izraz (19) lahko vpliv okna zmanjšamo z zmanjšanjem parametra σ. Tega lahko zmanjšamo do približno $\sigma_{1\mathrm{Hz}}$ = 1,5 [19], vendar pa to ni edina omejitev, pri zmanjševanju moramo biti pazljivi tudi na premik frekvenčnega vrha [9]:

3.1 Reduced-time spread of the wavelet function method – RTS

With the help of the parameter σ of the Gabor wavelet function we can easily control the desired time or frequency spread. The minimum value of the parameter can be approximately $\sigma_{1\mathrm{Hz}}$ = 1.5 [19]. However, we must have in mind the frequency shift of the ridge [9]:

$$\Delta\omega(s,\sigma,\eta)=\frac{\eta-\sqrt{2/\sigma^2+\eta^2}}{2\,s} \quad (20)$$

in na povečan frekvenčni raztros valčne funkcije; to pomeni da moramo bolj paziti, da se sosednje lastne frekvence ne motijo (13).

and the higher frequency spread of the wavelet function. Consequently we have to pay more attention to the close modes (13).

3.2 Metoda zrcaljenja okna – ZO

Gre za metodo, ki je podobna metodi zrcaljenja signala [19], vendar tukaj zrcalimo okno valčne

3.2 The reflected window method – RFW

This method tries to improve the symmetry of the non-symmetrical window on the edge [10]. The transformacije. Če vpliva roba ne upoštevamo, potem izračun delamo z nesimetrično valčno funkcijo, kar je v nasprotju s pogojem, da bi naj valčna funkcija bila simetrična [10]. Z zrcaljenjem to nesimetričnost izničimo. Izraz za izračun valčne transformacije je:

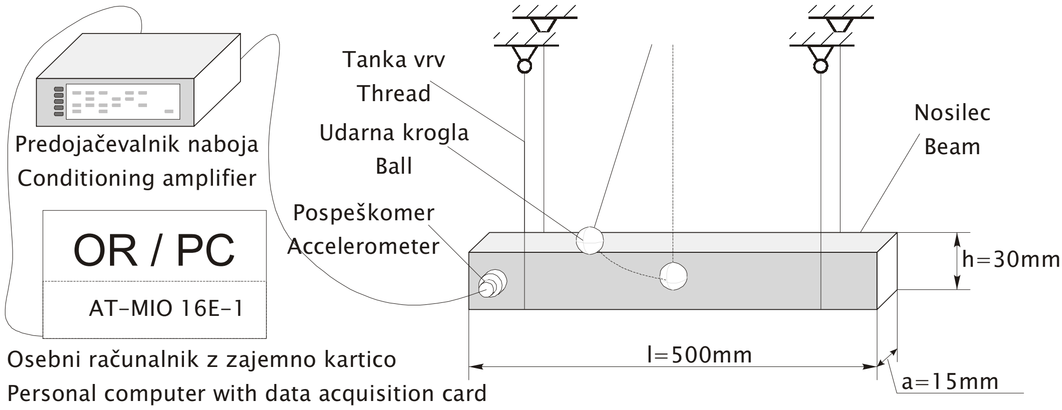
result is similar to the signal reflection [8]. The definition of the modified continuous wavelet transform is:

$$W_{\mathrm{RFW}} f(u,s) = \int_{-\infty}^{+\infty} f(t)\, \psi^{*}_{\mathrm{RFW}_{u,s}}(t)\, dt \qquad (21),$$

kjer je T dolžina signala. Spremenjena Gaborjeva valčna funkcija je:

where $\psi_{\mathrm{RFW}_{u,s}}$ is the modified Gabor wavelet function:

$$\psi_{\mathrm{RFW}_{u,s}}(t,\sigma,\eta) = \frac{1}{\sqrt{s}} \frac{1}{(\pi\sigma^2)^{1/4}} \left(e^{-\left(\frac{t-u}{\sqrt{2}s\sigma}\right)^2} + e^{-\left(\frac{t+u}{\sqrt{2}s\sigma}\right)^2} + e^{-\left(\frac{t-(2T-u)}{\sqrt{2}s\sigma}\right)^2} \right) e^{i\eta\frac{t-u}{s}} \qquad (22).$$



Sl. 1. *Shema preskusa*
Fig. 1. *Experimental set-up*

3.3 Metoda enakovredne površine okna – EPO

Pri tej metodi poskušamo nadomestiti del signala, ki manjka tako, da povečamo vpliv tistega, ki ga imamo na voljo. Kot merilo povečanja vzamemo delež površine okna znotraj signala.

Na začetku signala (časovno gledano) je ta delež enak:

3.3 The equal window area method – EWA

With this method we try to keep the proportionality of the absolute value of the wavelet transform. On the edge a part of the window is outside the signal, therefore we increase the value of the wavelet transform by the fraction between the whole window area and the window area inside the signal.

At the beginning of the signal the part of the window inside the signal is:

$$P_0(u,s,\sigma) = \frac{\int_{0}^{+\infty} |\psi_{\mathrm{Gabor}_{u,s}}(t,\sigma,\eta)|\, dt}{\int_{-\infty}^{+\infty} |\psi_{\mathrm{Gabor}_{u,s}}(t,\sigma,\eta)|\, dt} = \frac{1+\mathrm{Erf}\left(\frac{u}{\sqrt{2}s\sigma}\right)}{2} \qquad (23)$$

in na koncu (v okolici časa T):

and at the end of the signal (time T):

$$P_T(u,s,\sigma) = \frac{\int_{-\infty}^{+T} |\psi_{\mathrm{Gabor}_{u,s}}(t,\sigma,\eta)|\, dt}{\int_{-\infty}^{+\infty} |\psi_{\mathrm{Gabor}_{u,s}}(t,\sigma,\eta)|\, dt} = \frac{1+\mathrm{Erf}\left(\frac{T-u}{\sqrt{2}s\sigma}\right)}{2} \qquad (24).$$

Izraz za izračun valčne transformacije je:

The definition of the modified continuous wavelet transform is:

$$W_{\mathrm{EWA}} f(u,s) = P_0(u,s,\sigma)^{-1}\, P_T(u,s,\sigma)^{-1} \int_{-\infty}^{+\infty} f(t)\, \psi^{*}_{u,s}(t)\, dt \qquad (25).$$

4 PRESKUS

Cilj preskusa je bil določiti dušenje nevpetega nosilca (ρ = 7850 kg/m^3, E = 2,1.10^5 MPa) (sl. 1). Upogibno nihanje smo vzbudili s sunkovito motnjo. Čas zajemanja je bil 1/8 s, zajeli smo 2^{16} točk.

4 EXPERIMENT

The presented procedures were tested on a measured signal from the lateral vibrations of a uniform free-free beam (ρ=7850 kg/m^3, E=2,1.10^5 MPa). Figure 1 shows the experimental set-up. The length of the impulse response was 1/8 s, and the number of discrete points: 2^{16}.

Za dušenje nosilca smo uporabili model histereznega dušenja. Z uporabo modela enakovrednega viskoznega dušenja lahko predstavljene postopke uporabimo brez sprememb. Povezava med razmernikom dušenja in faktorjem histereznega dušenja je [20]: $\zeta=\eta/2$, kjer smo z η označili faktor histereznega dušenja.

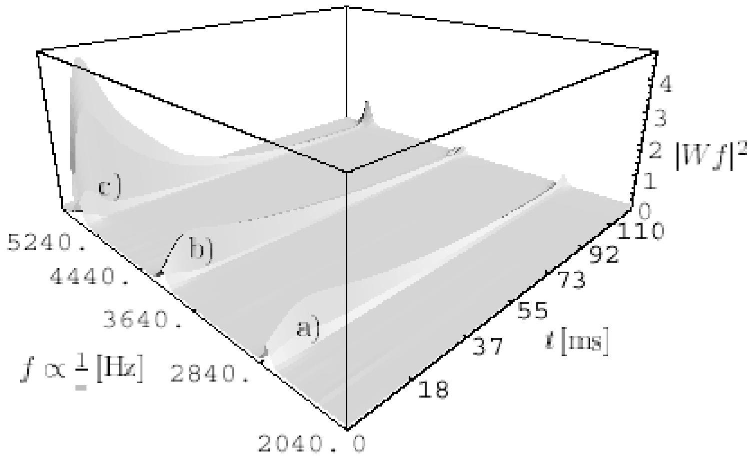
Tak nosilec smo že analizirali ([9], [21] in [22]), vendar nam je takrat uspelo razpoznati dušenje samo prvih treh lastnih frekvenc ($\eta_1=240\cdot10^{-6}$, $\eta_2=330\cdot10^{-6}$, $\eta_3=780\cdot10^{-6}$). Tukaj bomo poiskali še razmernike dušenja za 4., 5. in 6. lastno frekvenco.

Slika 2 prikazuje skalogram izmerjenega odziva, kjer so prikazane samo frekvence od 2000 Hz do 6000 Hz.

The model of hysteretic damping was used. The connection between the damping factor and the hysteretic damping factor η is defined by [20]: $\zeta=\eta/2$.

The damping features of the first three natural frequencies were already analyzed ([9], [21] and [22]). These damping factors are: $\eta_1=240\cdot10^{-6}$, $\eta_2=330\cdot10^{-6}$, $\eta_3=780\cdot10^{-6}$. In this work we will identify the damping of the further three higher natural frequencies (4th, 5th and 6th).

In Figure 2 the scalogram of the measured signal in the scale/frequency range from 2000 Hz up to 6000 Hz is shown.



Sl. 2. *Izmerjeni odziv v frekvenčnem področju od 2000 Hz do 6000 Hz. a) 4. lastna frekvenca, b) 5. lastna frekvenca, c) 6. lastna frekvenca*

Fig. 2. *Measured signal in the frequency range from 2000 Hz up to 6000 Hz. a) 4th natural frequency, b) 5th natural frequency, c) 6th natural frequency*

Postopek za oceno dušenja je:

- določitev dušene lastne frekvence (npr. s klasično diskretno Fourierjevo transformacijo),
- določitev grebena ZVT z metodo prečnega prereza (več metod v [9]), določitev ogrodja ZVT,
- izračun ovojnice odziva (17),
- določitev razmernika dušenja (15).

V preglednici 2 so zbrani pomembnejši podatki razpoznave dušenja za 4. lastno obliko. Opazimo, da je vpliv roba pri parametru k=6 širok 27,3 ms (na vsaki strani). Ker je signal dolg 125 ms, pri referenčni metodi [9] ostane še 70,4 ms koristnega signala (sl. 3a).

Logaritmi amplitude valčne transformacije (17) 4. lastne frekvence glede na izračun po različnih metodah so prikazani na sliki 3.

Rezultati razpoznave dušenja s predstavljenimi postopki, ki vpliv roba zmanjšajo tako, da lahko razpoznavo opravljamo na celotni dolžini signala 125 ms, so povzeti v preglednici 3. Opazimo, da so predlagane izboljšave primerno orodje za zmanjševanje vpliva roba na razpoznavo dušenja.

Tukaj je še pomembno opozoriti na dejstvo, da metoda zmanjšanega časovnega raztrosa valčne funkcije bistveno poveča frekvenčni raztros. V

The procedure for the damping identification is as follows:

- extract the natural frequency (can be done with a classical Fourier transform),
- extract the ridge with the cross-sections method (further methods in [9]), extract the skeleton,
- calculate the envelope (equation (17)),
- extract the damping ratio (equation (15)).

In Table 2 the parameters used in identifying the damping of the 4th natural frequency are shown. When the parameter k = 6 is used the radius of trust is 27.3 ms. If we use the reference method [9] we have only 70.4 ms of the 125 ms of the wavelet transform left to identify the damping, see Figure 3a.

The results of equation (17) for the different methods – RTS, EWA, and RFW – for the 4th natural frequencies are shown in Figure 3.

The identification of the damping with the enhanced methods was calculated on the whole length of the signal (125 ms). The results are given in Table 3. We can conclude that the new methods can be used as a substitute for the reference method.

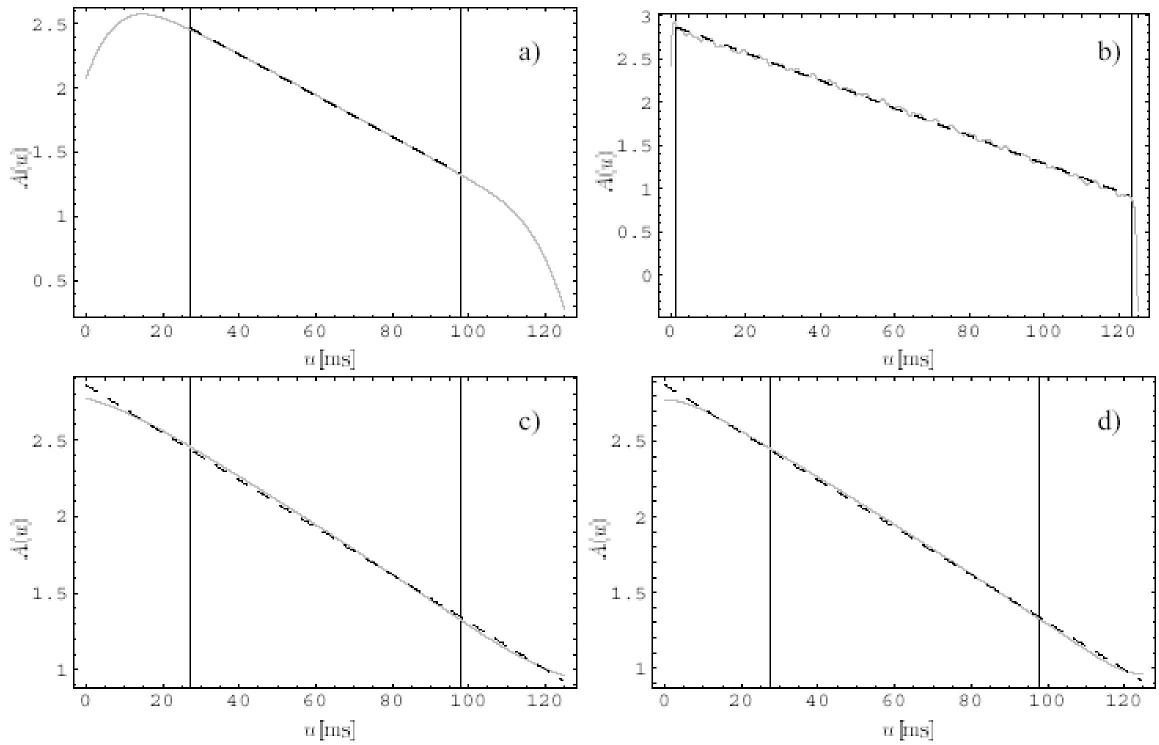
We must point out that the reduction of the time width used in the RTS method increases the

Preglednica 2. *Razpoznava dušenja 4. lastne oblike*
Table 2. *Identification of damping of the 4th natural frequency*

Parameter	Ref.	ZČR / RTS	EPO / EWA	ZO / RFW
η [Hz]		411775		
σ_{1Hz}	25	1,5	25	25
R [ms]	27,3	1,7	27,3	27,3
$\Delta\omega$ [Hz]	70	1167	70	70

Preglednica 3. *Pregled dušenja 4., 5. in 6. lastne frekvence*
Table 3. *Overview of the damping factors of 4th, 5th and 6 th natural frequency*

Lastna frek./Natural freq.	ω_d [Hz]	Ref.	ZČR / RTS	EPO / EWA	ZO / RFW
4.	2752	$1856{\cdot}10^{-6}$	$1880{\cdot}10^{-6}$	$1792{\cdot}10^{-6}$	$1817{\cdot}10^{-6}$
5.	4056	$941{\cdot}10^{-6}$	$962{\cdot}10^{-6}$	$934{\cdot}10^{-6}$	$942{\cdot}10^{-6}$
6.	5592	$1408{\cdot}10^{-6}$	$1434{\cdot}10^{-6}$	$1405{\cdot}10^{-6}$	$1412{\cdot}10^{-6}$



Sl. 3. *Logaritem amplitude (17) za 4. lastno frekvenco. - - - navpične črte označujejo področje vpliva roba (k=6)*
Fig. 3. *Logarithm of the amplitude (17) for the 4th natural frequency. - - - vertical lines denote the radius of trust (k=6).*
a) Ref., b) ZČR / RTS, c) EPO / EWA, d) ZO / RFW.

preglednici 2 vidimo, da se v primeru razpoznave 4. lastne frekvence le-ta poveča s 70 na 1167 Hz. To pomeni določeno omejitev, saj se lahko sosednje lastne frekvence v frekvenčnem prostoru medsebojno prekrivajo (13).

5 SKLEP

Prispevek je nadgradnja našega preteklega dela [9], v katerem smo podrobneje predstavili različne načine določevanja grebena za razpoznavo dušenja z uporabo zvezne valčne transformacije: metodo

frequency spread. The frequency spread at the 4th natural frequency of the RTS method is 1167 Hz, while the frequency spread of the other methods is 70 Hz, see Table 2. However, the 4th natural frequency does not interfere with the 3th and 5th (13).

5 CONCLUSION

This paper continues our previous efforts on damping identification [9] where we have studied different methods for ridge detection: the cross-sections method, the amplitude method and the phase prečnega prereza, amplitudno metodo in fazno metodo. V omenjenem delu smo razpoznavo izvajali samo na področju, kjer je vpliv roba zanemarljiv. Ker pa to ni vedno mogoče, smo tukaj predstavili tri izboljšane metode za razpoznavo dušenja, ki temeljijo na zmanjšanju vpliva roba: metodo zmanjšanega časovnega raztrosa valčne funkcije, metodo enakovredne površine okna in metodo zrcaljenja okna.

Na preskusu smo pokazali, da so nove metode, ki so učinkovite tudi na robu, primerna zamenjava za primerjalno metodo, ki je na področju robu neučinkovita. Med rezultati posameznih metod in primerjalno metodo nismo zaznali večjih odstopanj.

Metoda zmanjšanega časovnega raztrosa valčne funkcije ima to pomanjkljivost, da povečuje frekvenčni raztros valčne funkcije, kar pomeni, da moramo paziti na prekrivanje lastnih frekvenc v frekvenčnem prostoru.

Med metodo enakovredne površine okna in metodo zrcaljenja okna nismo zaznali bistvenih razlik, vendar pa velja omeniti, da je slednja numerično zahtevnejša.

Rezultati preskusa se razmeroma dobro ujemajo s podatki iz literature [23].

method. While in previous studies we identified the damping on the part of the CWT that was not effected by edge-effect, we introduced here three new, enhanced methods for the enhanced damping identification: the reduced time spread of the wavelet function method, the reflected window method and the equivalent window area method.

The tests of these new methods showed that they can be used as a good substitute for the old method. We have not detected any significant differences in the identification quality between the reference/old method and the new methods that proved to be efficient on the edge.

The reduced time width of the wavelet-function method is very efficient in reducing the edge effect, but it increases the frequency spread, therefore it is not appropriate for the identification of damping in the presence of close modes.

The differences in the identification of damping between the equivalent window area method and the reflected window method are small; however, the reflected window method demands more computer resources.

The experimental results are in good agreement with those found in the literature [23].

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Naslov avtorjev: Janko Slavič
doc.dr. Miha Boltežar
Univerza v Ljubljani
Fakulteta za strojništvo
Aškerčeva 6
1000 Ljubljana
janko.slavic@fs.uni-lj.si
miha.boltezar@fs.uni-lj.si

Authors' Address: Janko Slavič
Doc.Dr. Miha Boltežar
University of Ljubljana
Faculty of Mechanical Eng.
Aškerčeva 6
1000 Ljubljana, Slovenia
janko.slavic@fs.uni-lj.si
miha.boltezar@fs.uni-lj.si

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