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Are superheavy quark clusters viable candidates for the dark matter?*

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Abstract. The ordinary matter, as we know it, is made mostly of the first family quarks and leptons, while the theory together with experiments has proven so far that there are (at least) three families. The explanation of the origin of families is one of the most promising ways to understand the assumptions of the *Standard Model*. The *Spin-Charge-Family* theory [1,2] does propose the mechanism for the appearance of families which bellow the energy of unification scale of the three known charges form two decoupled groups of four families. The lightest of the upper four families, is predicted [3] to have stable members and to be the candidate to constitute the dark matter. The clustering of quarks from the fifth family into baryons in the evolution of the universe is discussed.

In this contribution we study how much the electroweak interaction influences the properties of baryons of the fifth family.

1 Introduction

The *Standard Model* has no explanation for either the existence of families and their properties or for the appearance of the scalar field, which in the *Standard Model* determines the properties of the electroweak gauge fields. A theory which would explain the origin of families and the mechanism causing the observed properties of the quarks, leptons and gauge fields is needed. The *Spin-Charge-Family* theory [1,2] is very promising for this purpose.

The *Spin-Charge-Family* theory points out that there are two kinds of the γ^a operators, the Dirac ones and the ones observed by one of the authors (NMB) [1,2] and called $\tilde{\gamma}^a$, and it proposes that both should appear in an acceptable theory (or it should be proven that one of these two kinds has no application at the observable energy regime). Since the operators $\tilde{\gamma}^a$ and γ^b anticommute, while the corresponding generators of rotations in d-dimensional space commute ($[S^{ab}, \tilde{S}^{cd}] = 0$), both kinds form equivalent representations with respect to each other. If Dirac operators are used to describe spin and charges [1,2], then the other kind must be used to describe families, which obviously form an equivalent representations with respect to spin and charges.

* Delivered in two talks, by M. Rosina and by N. Mankoč Borštnik

The properties of the fifth family quarks and leptons, and corresponding baryons, have been evaluated in ref. [3], concluding that the fifth family neutron is very probably the most stable nucleon. In this paper, the formation of neutrons and anti-neutrons from the fifth family quarks and anti-quarks in the cooling plasma has been followed in the expanding universe. Their behaviour in the colour phase transition up to the present dark matter, as well as the scattering of the fifth family neutrons among themselves and on the ordinary matter has been evaluated.

The purpose of this contribution is to show an example how one can use standard hadronic calculations in order to examine possible higher families and candidates for dark matter. It is also a demonstration of how much the properties of clusters depend on the masses of the objects forming the clusters.

We shall use the promising unified *Spin-Charge-Family* theory [1] which has been developed by one of the authors (NMB) in the recent decade. The reader can find details about the theory in the references [1], while Sect. 2 is a short overview, needed for the purpose of this contribution.

Let us remind the reader about possible prejudices one might have at the first moment against accepting the particles which interact with the colour interaction, as candidates for dark matter. We discuss these prejudices in order to demonstrate that superheavy quark clusters are legitimate candidates worth exploring, provided they are stable.

1. *Superheavy quarks are too short-lived.* This is true for the fourth family predicted by the *Spin-Charge-Family* theory, or any other proposal if the mixing matrix elements to the lower mass families are not negligible. However, the *Spin-Charge-Family* theory [1,4] predicts eight families, with the upper four families (almost) decoupled from the lower four families. This makes one of the quarks of the fifth family, actually one of possible baryonic clusters, practically stable.
2. *Either the charged baryon $u_5u_5u_5$ or the charged baryon $d_5d_5d_5$ would be the lightest, depending on whether u_5 or d_5 is lighter.* Charged clusters cannot, of course, constitute dark matter. Forming the atoms with the first family electrons they would have far too large scattering amplitude to be consistent with the properties of dark matter. However, if one takes into account also the electro-weak interaction between quarks, then the neutral baryon $n_5 = u_5d_5d_5$ can very probably be the lightest, provided the u-d mass difference is not too large. The ref. [3] estimates the allowed differences, here we present the ratio between the weak and electromagnetic contributions for different fifth family baryons in more detail (Sect. 3).
3. *Strongly interacting particles have far too large cross section to be "dark".* The scattering cross section of any neutral cluster due to any interaction depends strongly also on the mass of the constituents. The fifth family baryons, interacting with the fifth family "nuclear force", have very small cross section if the masses are large enough. For $m_5 = 100$ TeV, for example, the size of the cluster is of the order 10^{-4} fm or less and the geometrical cross section as small as 10^{-10} fm^2.

4. *Did the fifth family quarks and/or their clusters form and survive after the big bang and during galaxy formation?* We kindly invite the reader to learn about the history of the fifth family clusters in the expanding universe from the paper [3]. In a hot plasma, when the temperature T is much higher than the mass of the fifth family members, $T >> m_5$, the fifth family members behave as massless and are created out of plasma and annihilate back in the thermodynamically equilibrium in the same way as other fermions and fields, which are massless or have low enough masses. When due to the expansion of the universe the temperature lowers bellow the mass of the family members, $T < m_5$, they can be annihilated while the creation starts to be less and less probable. When the temperature falls bellow the binding energy of the clusters of the fifth family quarks they start to form clusters. Once the cluster is formed, it starts to interact with a very small "fifth family nuclear force" and survives also the colour phase transition up to now. In [3,5] the scattering of the fifth family neutrons in the experimental equipment of DAMA [6] and CDMS [7] is evaluated and discussed.

2 The Spin-Charge-Family theory

In this section a short introduction to the *Spin-Charge-Family* theory [1] is presented. Only the essential things are reviewed hoping to make the reader curious to start thinking about the differences in the hadronic properties of the very heavy fifth family hadrons as compared to the lowest three families.

The *Spin-Charge-Family* theory proposes in $d = (1 + (d - 1))$ dimensions a very simple starting action for spinors which carry both kinds of the spin generators (γ^a and $\tilde{\gamma}^a$ operators) and for the corresponding gauge fields. Multidimensional spinors unify the spin and electro-weak-colour charge degrees of freedom. A spinor couples in $d = 1 + 13$ to vielbeins and (through two kinds of the spin generators) to spin connection fields. Appropriate breaking of the starting symmetry leads to the left-handed quarks and leptons in $d = (1 + 3)$ dimensions, which carry the weak charge while the right handed ones carry no weak charge. The *Spin-Charge-Family* theory is offering the answers to the questions about the origin of families of quarks and leptons, about the explicit values of their masses and mixing matrices, predicting the fourth family to be possibly seen at the LHC or at somewhat higher energies [4], as well as about the masses of the scalar and the weak gauge fields, about the dark matter candidates [3], and about breaking the discrete symmetries.

The simple action in $d = (1+13)$-dimensional space of the *Spin-Charge-Family* theory [1]

$$S = \int d^d x \, E \, \mathcal{L}_f + \int d^d x \, E \, \mathcal{L}_g \tag{1}$$

contains the Lagrange density for two kinds of gauge fields linear in the curvature

$$\begin{aligned}\mathcal{L}_g &= E\,(\alpha\, R + \tilde{\alpha}\tilde{R}),\\ R &= f^{\alpha[a}f^{\beta b]}\,(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\omega^{c}{}_{b\beta}),\ \tilde{R} = f^{\alpha[a}f^{\beta b]}\,(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^{c}{}_{b\beta}),\end{aligned} \tag{2}$$

and for a spinor, which carries in $d = (1 + 13)$ dimensions two kinds of the spin represented by the two kinds of the Clifford algebra objects [1]

$$S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a), \quad \tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a),$$
$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \quad \{S^{ab}, \tilde{S}^{cd}\}_- = 0. \tag{3}$$

The interaction is only between the vielbeins and the two kinds of spin connection fields

$$\mathcal{L}_f = \frac{1}{2}(E\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.}$$
$$p_{0a} = f^\alpha{}_a p_{0\alpha}, \quad p_{0\alpha} = p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}. \tag{4}$$

This action offers a real possibility to explain the assumptions of the *standard model* [1].

The *Spin-Charge-Family* theory predicts an even number of families, among which is the fourth family, which might be seen at the LHC [1,4] or at somewhat higher energies and the fifth family with neutrinos and baryons with masses of several hundred TeV forming dark matter [4].

The action in Eq. (1) starts with the massless spinor which through two kinds of spins interacts with the two kinds of the spin connection fields. The Dirac kind of the Clifford algebra objects (γ^a) determines, when the group $SO(1,13)$ is analysed with respect to the *Standard Model* groups in $d = (1+3)$ dimensions, the spin and all charges, manifesting the left handed quarks and leptons carrying the weak charge and the right handed weak-neutral quarks and leptons. Accordingly, the Lagrange density $\mathcal{L}_f$ manifests after the appropriate breaking of symmetries all the properties of one family of fermions as assumed by the *Standard Model*, with the three kinds of charges coupling fermions to the corresponding three gauge fields (first term of Eq.(5).

The second kind ($\tilde{\gamma}^a$) of the Clifford algebra objects (defining the equivalent representations with respect to the Dirac one) determines families. Accordingly, the spinor Lagrange density, after the spontaneous breaking of the starting symmetry ($SO(1,13)$ into $SO(1,7) \times U(1) \times SU(3)$ and further into $SO(1,3) \times SU(2) \times SU(2) \times U(1) \times SU(3)$) generates the *Standard Model-like* Lagrange density for massless spinors of (four + four) families (defined by $2^{8/2-1} = 8$ spinor states for each member of one family). After the first symmetry breaking the upper four families decouple from the lower four families (in the Yukawa couplings). In the final symmetry breaking (leading to $SO(1,3) \times U(1) \times SU(3)$) the upper four families obtain masses through the mass matrix (the second term of Eq.(5). The third term ("the rest") is unobservable at low energies

$$\mathcal{L}_f = \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A\tau^{Ai}A^{Ai}_m)\psi + \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi + \text{the rest}. \tag{5}$$

[1] This is the only theory in the literature to our knowledge, which does not explain the appearance of families by just postulating their numbers in one or another way, through the choice of a group, for example, but by offering the mechanism for generating families.

Here τ^{Ai} $(= \sum_{a,b}\ c^{Ai}{}_{ab}\ S^{ab})$ determine the hypercharge $(A = 1)$, the weak $(A = 2)$ and the colour $(A = 3)$ charge: $\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}$, $f^{1ijk} = 0$, $f^{2ijk} = \varepsilon^{ijk}$, where f^{3ijk} is the $SU(3)$ structure tensor.

The evaluation of masses and mixing matrices of the lower four families [4] suggests that the fifth family masses should be above a few TeV, while evaluations of the breaks of symmetries from the starting one (Eq. 1) suggests that these masses should be far bellow 10^{10} TeV.

We have not yet evaluated a possible fermion number nonconservation in the dynamical history of the universe either for the first (the lower four) or for the fifth (the upper four) families. However, the evaluation of the history of the fifth family baryons up to today's dark matter does not depend much on the matter anti-matter asymmetry, as long as the masses are higher than a few 10 TeV. So our prediction that if DAMA [6] really measures the family neutrons, also other direct experiments like CDMS [7] should in a few years observe the dark matter clusters, does not depend on the baryon number nonconservation [3].

Following the history of the fifth family members in the expanding universe up to today and estimating also the scattering properties of this fifth family on the ordinary matter, the evaluated masses of the fifth family quarks, under the assumption that the lowest mass fifth family baryon is the fifth family neutron, are in the interval

$$200\,\text{TeV} < m_5 < 10^5\,\text{TeV}. \tag{6}$$

The fifth family neutrino mass $m_{\nu 5}$ is estimated to be in the interval between a few TeV and a few hundred TeV.

3 The superheavy neutron from the fifth family as a candidate for the dark matter

We want to put limits on u-d quark mass differences so that the neutral baryon n_5 appears as the lightest. First we calculate the dominant properties of a three-quark cluster [3], its binding energy and size. For this purpose we assume equal superheavy masses and we realize that in this regime the colour interaction is coulombic (one gluon exchange dominates at these energies). For three nonrelativistic particles with attractive coulombic interaction we solve the Hamiltonian

$$H = 3m + \sum_i \frac{\mathbf{p}_i^2}{2m} - \frac{(\sum_i \mathbf{p}_i)^2}{6m} - \sum_{i<j} \frac{2}{3}\frac{\alpha_s}{r_{ij}}. \tag{7}$$

The potential energy of the solution can be conveniently parametrized as

$$V_s = -\frac{2}{3}\alpha_s \epsilon, \quad \epsilon = \langle \sum_{i<j} \frac{1}{r_{ij}} \rangle = 3\eta\alpha_s m_5, \tag{8}$$

where m_5 is the average mass of quarks in the fifth family. The binding energy is then (according to the virial theorem)

$$E = \frac{1}{2} V_s = -E_{kin} = -\alpha_s^2 \eta m_5. \tag{9}$$

The parameter η for a variational solution using Jacobi coordinates and exponential profiles was calculated in [3]: $\eta = 0.66$.

The splitting of baryons in the fifth family is caused by the u-d mass difference as well as by the potential energy of the electro-weak interaction. In the studied energy range, the electro-weak interaction has a coulombic form, determined by the exchange of one photon or one massless weak boson, and can be treated as a perturbation. Even if we are far above the electroweak phase transition, it is convenient to work in the basis using Weinberg mixing of γ and Z since this basis is more familiar to low energy hadron physicists.

We split the electro-weak interaction in five contributions, electric, Z-exchange Fermi (=vector), Z-exchange Gamow-Teller (=axial), W-exchange Fermi (=vector), W-exchange Gamow-Teller (=axial)

$$M = \sum_i m_i + E + \left(V_{\mathrm{EM}} + V_Z^{\mathrm{F}} + V_Z^{\mathrm{GT}} + V_W^{\mathrm{F}} + V_W^{\mathrm{GT}}\right) . \tag{10}$$

Separate terms are as follows

$$\begin{aligned}
V_{\mathrm{EM}} &= \langle \sum_{i<j} Q_i Q_j \rangle\, \alpha_{\mathrm{EM}} \epsilon, \\
V_Z^{\mathrm{F}} &= \langle \sum_{i<j} (\frac{t_i^0}{2} - \sin^2\vartheta_W Q_i)(\frac{t_j^0}{2} - \sin^2\vartheta_W Q_j) \rangle\, \alpha_Z \epsilon,\ V_Z^{\mathrm{GT}} = \langle \sum_{i<j} \frac{t_i^0 t_j^0}{4} \boldsymbol{\sigma}_i \boldsymbol{\sigma}_j) \rangle\, \alpha_Z \epsilon, \\
V_W^{\mathrm{F}} &= \langle \sum_{i<j} \frac{t_i^- t_j^+ + t_i^+ t_j^-}{8} \rangle\, \alpha_W \epsilon, \quad V_W^{\mathrm{GT}} = \langle \sum_{i<j} \frac{t_i^- t_j^+ + t_i^+ t_j^-}{8} \boldsymbol{\sigma}_i \boldsymbol{\sigma}_j \rangle\, \alpha_W \epsilon.
\end{aligned} \tag{11}$$

Here $\mathbf{t} = \frac{1}{2}\boldsymbol{\tau}$ are isospin operators, $t^+ = (t_x + t_y)$, and $\boldsymbol{\sigma}$ are Pauli spin matrices. Separate terms are evaluated in Table 1. Note that the vector contributions (also the electromagnetic) are the same for N and Δ baryons while the axial contributions differ dramatically. The lowest two lines give the sum of these contributions for the choice of the coupling constants given below. The unnecessary decimal places are there if you like to check the reproducibility of the results.

In the numerical example we choose the average quark mass m_5 = 100 TeV and the corresponding average momentum of each quark $p = \sqrt{2m_5\, E_{\mathrm{kin}}/3} =$ 5.1 TeV (see below). At this momentum scale, we read the running coupling constants from Particle Data Group diagram [8] as $\alpha_s = \alpha_3 = 1/13$, $\alpha_W = \alpha_2 = 1/32$ and $\alpha_1 = 1/56$. The latter gives $\sin^2\vartheta_W = (1 + \frac{5}{3}\frac{\alpha_W}{\alpha_1})^{-1} = 0.255 \approx 1/4$, $\alpha_{\mathrm{EM}} = \alpha_W \sin^2\vartheta_W = 1/128$ and $\alpha_Z = \alpha_W / \cos^2\vartheta_W = 1/24$.

In this example, the binding energy $E = -0.39$ TeV and the average reciprocal distance $\langle 1/r_{ij} \rangle = \epsilon/3 = \eta\alpha_s m_5 = 5.1\,\mathrm{TeV} = 2.6 \cdot 10^4\,\mathrm{fm}^{-1}$.

Finally, we come to our goal to make limits on u-d mass difference such that the neutral barion remains the lightest.

1. $m_{u5} - m_{d5} < (0.0273 - 0.0017)\epsilon = 0.0256\,\epsilon$ prevents udd$\to$ddd.
2. $m_{u5} - m_{d5} > (-0.0273 + 0.0256)\epsilon = -0.0017\,\epsilon$ prevents udd$\to$uud.

For our value of $\epsilon = 15.24$ TeV this reads

$$-0.026\,\mathrm{TeV} < m_{u5} - m_{d5} < 0.39\,\mathrm{TeV}.$$

Table 1. Electro-weak contributions to superheavy baryon masses

	uuu	uud	udd	ddd
$V_{EM}/\epsilon\alpha_{EM}$	+4/3	0	−1/3	+1/3
$V_Z^F/\epsilon\alpha_Z$	+1/48	−1/48	0	+4/48
$V_Z^{GT}(N)/\epsilon\alpha_Z$		−15/48	−15/48	
$V_Z^{GT}(\Delta)/\epsilon\alpha_Z$	−9/48	+3/48	+3/48	−9/48
$V_W^F/\epsilon\alpha_W$	0	+1/4	+1/4	0
$V_W^{GT}(N)/\epsilon\alpha_W$		−30/48	−30/48	
$V_W^{GT}(\Delta)/\epsilon\alpha_W$	0	−1/4	−1/4	0
$V_{EW}(N)/\epsilon$		−0.0256	−0.0273	
$V_{EW}(\Delta)/\epsilon$	+0.0035	+0.0017	−0.0000	−0.0017

This limits are narrow compared to the mass scale $m_5 = 100$ TeV, but they are not so narrow if the mass generating mechanism is of order of 100 GeV.

4 Conclusion

In this contribution we put light on the hadronic properties of the very heavy stable fifth family as predicted by the *Spin-Charge-Family* theory, proposed by one of the authors [1]. The evaluations presented in Sect. 3 were already partially done in [3]. However, we try to convince the hadron physicists that if the *Spin-Charge-Family* theory is the right way to explain the assumptions of the *Standard Model* then the hadron physicists will have a pleasant time to study properties of the clusters forming dark matter with their knowledge form the lower three families.

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