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7 Initial Condition From the Action Principle, Its Application To Cosmology and To False Vacuum Bubbles

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Abstract. We study models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe. We first consider the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and afterwards extend this scalar fields. For Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field. In the case where scalar field are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the Higgs field, the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. Also false vacuum branes will be studied in a similar approach. We discuss also the use of ″spoiling terms″, that violate gauge invariance to introduce these initial condition.

Povzetek. Obravnavava modele, v katerih so umeritvene sklopitvene konstante, mase, itd. funkcije nekega naboja, ki se v vesolju ohranja. Najprej študirava običajno Diracovo akcijo, v kateri so mase in elektromagnetna sklopitev funkcije tega naboja. Za Diracovo polje v ravnem prostoru akcija ni manifestno Lorentzovo invariantna, vendar za invarianco lahko poskrbimo s fazno transformacijo Diracovega polja. Če pa zamenjava fermione s skalarnimi polji, lahko iz akcije izpeljemo tudi začetni pogoj. Za Higgsovo polje začetni pogoj zahteva, da je vesolje ob določenem časovnem intervalu (časovni rezini) v lažnem vakuumu, kar omogoči različne možnosti za inflacijo vesolja. S podobnim pristopom se bova lotila tudi bran lažnega vakuuma. Na koncu obravnavava še uporabo „kvarnih členov", ki kršijo umeritveno invarianco, ker ponudijo primerne začetne pogoje.

7.1 Introduction

Landau said ″ The future physical theory should contain not only the basic equations but also the initial conditions for them ″ [1]. In physics we deal with equation of motion that are obtained by varying the action, here the question of the initial condition or boundary condition are normally separated from the equation of motion, and by giving them both we can solve the physical problem (like in many differential equation problems where the solution is determined by the initial condition). Knowing just the equation of motion or just the initial conditions does not give the solution of the problem. From this point we are motivated to construct a model where initial conditions can be found from the fundamental rules of physics, without the need to assume them, they will be derived. Also we want to check whether the new model is consistent with causality and other requirements. One of the examples of a system where the initial conditions are indirectly known, and the question is why should the initial condition be like that is the inflaton model. Today there are many models for inflation, the models are defined by the kind of inflation potential. The question is why the initial field should have specific initial conditions.

The problem in the inflation initial condition is that there is not known proven way to start the universe from a false vacuum state with vacuum energy density higher than the present universe needed for inflation. In fact it appears counter intuitive not to start in the lower energy state. One idea, the "eternal inflation" that one may think solve the problem, in fact does not solve the problem. Guth et.al wrote in their paper [2] "*Thus inflationary models require physics other than inflation to describe the past boundary of the inflating region of space time*". In their article it was proven that in the past of the eternal inflationary model there must be a singularity. Also there are some initial singularity problems related to creation of baby universe from false vacuum, such a singularity cannot conceivably be produced in the laboratory, since it has no prior history [8], so we need some reason for such initial condition for the singularity of the creation of the universe. Also we are motivated to consider another direction in the research and study a model where the boundary conditions can follow from the action, this kind of approach can be used in a model where space like boundary condition of a system are fixed without any additional assumption, therefore fixing false vacuum boundary conditions on a brane. There are some equations in mathematical physics that constrain the possible initial condition that one can give. For example in electrodynamics , the equation $\nabla \cdot E = 4\pi\rho$ is a time independent equation for E and ρ, but tell us that we cannot give an initial value problem where $\nabla \cdot E = 4\pi\rho$ is not satisfied. We want to deal in fact with a sort of constraint equations, but which do not impose a constraint every where, but only for a surface (time like or space like) therefore providing in fact initial or boundary condition, in the next section we review some ideas on actions whose couplings depend on charges [3] which will be the basis to achieve this, when charged scalar fields are introduced (following section). We will see that generalizing the models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe may give such effect.

In a previous publication we considered the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and in this work we extend this scalar fields. This was motivated by the idea of obtaining a Mach like principle. For the Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field. In the case where scalar field are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the

Higgs field inflation [5], the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. False vacuum branes will be studied in a similar approach.

7.2 The electromagnetic coupling constant as a function of the charge in the Dirac field

We begin by considering the action for the Dirac equation

$$S = \int d^4x \, \bar{\psi}(\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - eA_\mu\gamma^\mu - m)\psi \tag{7.1}$$

where $\bar{\psi} = \psi^\dagger\gamma^0$.However here we take the coupling constant e to be proportional to the total charge (It can be generalized and we can also consider an arbitrary function of the total charge[3]).

$$e = \lambda_e \int \psi^\dagger(\vec{y}, y^0 = t_0)\psi(\vec{y}, y^0 = t_0) \, d^3y = \lambda_e \int d^4y \, \bar{\psi}(y)\gamma^0\psi(y)\delta(y^0 - t_0) \tag{7.2}$$

and we will show that physics does not depend on the time slice $y^0 = t_0$. If we consider the fact that $\frac{\delta\bar{\psi}_a(x)}{\delta\bar{\psi}_b(z)} = \delta^4(x - z)\,\delta_{ab}$ and $\frac{\delta\psi(x)}{\delta\bar{\psi}(z)} = 0$ we get the equation of motion, where $b_e = \lambda_e(\int \bar{\psi}(x)A_\mu\gamma^\mu\psi(x)\, d^4x)$.

$$\frac{\delta S}{\delta\bar{\psi}(z)} = [i\gamma^\mu\partial_\mu - m - eA_\mu\gamma^\mu - b_e\gamma^0\delta(z^0 - t_0)]\psi(z) = 0 \tag{7.3}$$

so we can see that the last term in the equation of motion (7.3) contains $A^{GF}_\mu\gamma^\mu$ where $A^{GF}_\mu = \partial_\mu\Lambda$ and $\Lambda = b_e\theta(z^0 - t_0)$ is a pure gauge field. so the solution of this equation is

$$\psi = e^{-ib_e\theta(z^0 - t_0)}\psi_D \tag{7.4}$$

where ψ_D is the solution of the equation

$$[i\gamma^\mu\partial_\mu - m - eA_\mu\gamma^\mu]\psi_D = 0 \tag{7.5}$$

from which it follows that $j^\mu = \bar{\psi}_D\gamma^\mu\psi_D = \bar{\psi}\gamma^\mu\psi$ satisfies the local conservation law $\partial_\mu j^\mu = 0$ and therefore we obtain that $Q = \int d^3x\, j^0$ is conserved, so it does not depend on the time slice, furthermore it also follows that it is a scalar. For more examples see in referance [3]

7.3 Action which incorporates initials conditions

As we will see now that type of actions considered in the previous section, when generalizing them to include charged scalar fields can provide some initials condition for the scalar field [4]. Those actions can be produced by taking the coupling constants as a function of a conserved charge. If we use this development we can have the initial vacuum state for the universe in the inflationary model, so this initial condition will give us the initial condition for the universe corresponding to being initially at the false vacuum. Following there are some examples of actions that can produce initial conditions.

We will use the definition of the book of Anderson [10], which take the points in sub-manifold:

$$x^\mu = \Phi^\mu(\lambda_1, ..., \lambda_N) \tag{7.6}$$

the definition of the charge is:

$$\Theta = \lambda \int j_\mu d\sigma^\mu = \lambda \int j_\mu \delta^\mu(x - \Phi) d\sigma \tag{7.7}$$

where $d\sigma = \sqrt{-g} d^4x$ and $d\sigma^\mu$ is hyper-surface volume orthogonal to the normal n^μ. Φ^μ is parametric $d-1$ hyper-surface, and the delta function can be defined by $\delta^\mu = \int \delta^4(x - \Phi) d\sigma^\mu$ and the correspond step function are defined by $\theta = \int_x \delta(x - \Phi) d\sigma$. It is not hard to prove that Θ is constant [10]. The action for Higgs inflation with the general potential $V(\phi, \phi^*, \Theta)$ is[5]

$$\begin{aligned}
S = \int d\sigma\sqrt{-g}\,[(\partial_\mu\phi^* + i\tfrac{g'}{2}A_\mu\phi^*)(\partial^\mu\phi - i\tfrac{g'}{2}A^\mu\phi) - V(\phi, \phi^*, \Theta) + \tfrac{\alpha}{16\pi}\phi^*\phi R] \\
-\tfrac{1}{4}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d\sigma + \tfrac{M_p}{2}\int\sqrt{-g}R\,d\sigma = \\
\int d\sigma\sqrt{-g}\,[(D\phi)^*(D\phi) - V(\phi, \phi^*, \Theta) + \tfrac{\alpha}{16\pi}\phi^*\phi R] - \tfrac{1}{4}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d\sigma \\
+\tfrac{M_p}{2}\int\sqrt{-g}R\,d\sigma
\end{aligned} \tag{7.8}$$

from this by variation on ϕ^* , we get the equation of motion:

$$\begin{aligned}
-\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial^\nu\phi) - i\tfrac{g'}{2}\partial_\mu(\sqrt{-g}A^\mu\phi) - i\sqrt{-g}\tfrac{g'}{2}A_\mu\partial^\mu\phi \\
+\sqrt{-g}(\tfrac{g'}{2})^2A_\mu A^\mu\phi - \sqrt{-g}\tfrac{\partial V}{\partial\phi^*} + \tfrac{\alpha}{16\pi}\phi R \\
-\lambda\sqrt{-g}(\int d\sigma\,\sqrt{-g}\,\tfrac{\partial V}{\partial\theta})\delta^\mu(x - \Phi)[2i\partial_\mu\phi - g'A_\mu\phi] \\
-\lambda(\int d\sigma\,\sqrt{-g}\,\tfrac{\partial V}{\partial\theta})i\phi\,\partial_\mu(\sqrt{-g}\delta^\mu(x - \Phi)) = 0
\end{aligned} \tag{7.9}$$

We can see that we have delta term, so to eliminate it we do the transformation:

$$A^\mu \longrightarrow A^\mu + \frac{2i\lambda_1 b}{g'}\delta^\mu(x - \Phi) \tag{7.10}$$

and

$$\phi = e^{\lambda_2 b\theta(x-\Phi)}\phi_0 \tag{7.11}$$

where $b = i\lambda(\int d\sigma\,\sqrt{-g}\,\frac{\partial V}{\partial\theta})$
So we have that:

$$\begin{aligned}
-\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi_0) - i\tfrac{g'}{2}\partial_\mu(\sqrt{-g}A^\mu\phi_0) - i\sqrt{-g}\tfrac{g'}{2}A_\mu\partial^\mu\phi_0 \\
+\sqrt{-g}(\tfrac{g'}{2})^2A_\mu A^\mu\phi_0 - \sqrt{-g}\tfrac{\partial V}{\partial\phi^*} + \tfrac{\alpha}{16\pi}\phi R \\
-2b\sqrt{-g}\delta^\mu(x - \Phi)[(\lambda_2 - \lambda_1 + 1)\partial_\mu\phi_0 - i(\lambda_2 - \lambda_1 + 1)\tfrac{g'}{2}A_\mu\phi_0 \\
+0.5b\delta_\mu(x - \Phi)\phi_0(\lambda_2^2 - 2\lambda_1 + \lambda_1^2 + 2(-\lambda_1 + 1)\lambda_2)] \\
-b(\lambda_1 - \lambda_2 + 1)\phi_0\,\partial_\mu(\sqrt{-g}\delta^\mu(x - \Phi)) = 0
\end{aligned} \tag{7.12}$$

if we need that equation (7.12) will be like ordinary Klein Gordon equation we need that:

$$\lambda_2 - \lambda_1 + 1 = 0 \tag{7.13}$$
$$\lambda_2^2 - 2\lambda_1 + \lambda_1^2 + 2(-\lambda_1 + 1)\lambda_2 = 0 \tag{7.14}$$

for which there is no solution, so we must conclude that $\phi(\Phi) = 0$. We can take private case were the action give initial condition, were $\Theta = Q = \lambda \int j_0 \delta^0(t) d\sigma$, which give $\phi(t = 0) = 0$.

7.4 Boundary condition from spoiling terms

Some "spoiling terms" that is terms that break gauge invariance have been shown in the end do not to contribute the functional integral [11]. Here we will see that "spoiling terms" where non gauge invariant charge are introduced, have as a consequence that they induce boundary condition and these boundary condition imply the vanishing of the spoiling terms. To see this we take the action

$$\begin{aligned} S = \int d^4x\sqrt{-g}\,[(\partial^\mu\phi^* + i\tfrac{g'}{2}A^\mu\phi^*)(\partial_\mu\phi - i\tfrac{g'}{2}A_\mu\phi) - V(Q^{NGI}, \phi, \phi^*)] \\ -\tfrac{1}{4}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x - \tfrac{1}{16\pi G}\int \sqrt{-g}R\, d^4x \end{aligned} \tag{7.15}$$

where we introduce the non gauge invariant charge

$$Q^{NGI} = \int d^4x\sqrt{-g}\delta(t - t_0)[\phi^* i \overleftrightarrow{\partial^0} \phi + g_1' A^0\phi^*\phi] \tag{7.16}$$

where NGI= Non Gauge Invariant, and $g_1' \neq g'$. If we vary the action by a gauge transformation

$$A_\mu \to \partial_\mu\Lambda + A_\mu \tag{7.17}$$
$$\phi \to \phi_0 e^{ig'\Lambda} \tag{7.18}$$

all the other term of the action can not be change but

$$Q^{NGI} \to Q^{NGI} + (g' - g_1')\int d^4x\sqrt{-g}\delta(t - t_0)\partial_0\Lambda(x)\phi^*\phi \tag{7.19}$$

So for all $\Lambda(x)$ if $V(Q^{NGI}, \phi, \phi^*)$ has a non trivial dependence on Q^{NGI} then equating the variation of the action to zero implies:

$$\phi^*(t_0)\phi(t_0) = 0 \tag{7.20}$$

Of course, this means that the theory effectively cancels the non gauge invariant terms when the variational principle is used, so gauge invariance is restored effectively. Also boundary condition which are gauge invariant are obtained.

It is interesting to compare the mechanism obtained from introducing spoiling terms to the mechanism obtained in order to climb up a potential using ghost field and may be also in this way end up at the top of potential [12].

7.5 Discussions and Conclusions

We have studied models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe. We first considered the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and afterwards extended this to scalar fields. For Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field.

In the case where scalar fields are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the Higgs field, the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. Also false vacuum branes can be studied in a similar approach.

One should point out that it appears that not all possible boundary condition allow a formulation in this way, which is probably good, because we would like a theory of the boundary condition to restrict such possibilities.
Some ”spoiling terms” that is terms that break gauge invariance have been shown that in the end they do not contribute to the functional integral [11]. We have seen that ”spoiling terms” where non gauge invariant charges are introduced, have as a consequences that they induce boundary condition and these boundary condition imply the vanishing of the spoiling terms, and in the special example choose that the universe sits in the false vacuum in a certain time slice

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