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Prenos vibracij po ukrivljenih cevovodih v prostoru

Vibrations of a 3-Dimensional Piping System

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V prispevku je na kratko predstavljena metoda prenosnih matrik za modeliranje prenosa vibracij preko cevovoda v prostoru. Na temelju točne rešitve gibalnih enačb je na novo definirana prenosna matrika krožnega loka s tekočino, pri katerem upoštevamo tudi osno deformljivost. Osredotočili se bomo na ustaljeno stanje odziva ob harmonski vzbujevalni motnji. Poznavanje prenosne matrike ravne cevi in krožnega loka omogoča modeliranje prenosa vibracij po poljubnem cevovodu, kjer upoštevamo učinke pretakajoče se tekočine in nadtlaka v cevi. V numeričnem preizkusu je potrjeno pravilno delovanje metode na podlagi primerjave rezultatov prenosa sile, določene z metodo prenosnih matrik in metodo končnih elementov. Enako je potrjeno tudi s preizkusom. Izkaže se tudi, da je čas preračuna pri uporabi metode prenosnih matrik občutno krajši v primerjavi z metodo končnih elementov.

(Ključne besede: cevovodni sistemi, prenos vibracij, modeliranje, metode prenosnih matrik)

This paper briefly presents the transfer-matrix method (TMM) for the vibration analysis of a 3-dimensional piping system. The transfer matrix of a curved pipe with fluid was derived based on the exact solution of the governing equations, where the axial deformability was also taken into account. Only the steady-state response for the case of harmonic excitations was analyzed. With the formulation of the transfer matrix for a straight pipe and for a curved pipe, a 3-dimensional complex piping system can be easily modeled. The inviscid fluid-dynamic forces were derived according to the plug-flow approximation and the slender-body theory. As shown in a numerical example of a 3-dimesional piping system the comparison of the transfer function obtained with the finite-element method (FEM) and the TMM is quite good. On the basis of a comparison of the experimental results and the results obtained with the TMM, the correctness of the method was confirmed. It was also shown that the TMM is significantly faster than the FEM.

(Keywords: piping systems, vibration analysis, modeling, transfer-matrix method)

0 UVOD

V zadnjem času ima vedno več avtomobilov klimatske naprave, pri čemer so posamezni deli povezani s cevovodi. Ti cevovodi so izpostavljeni vibracijam kompresorja in vibracijam, ki se prenašajo po karoseriji. Nihajoči cevovod povzroča nezaželen hrup, prav tako pa lahko privede do odpovedi posameznih sklopov klimatskega sistema zaradi prenesenih vibracij. V izogib temu si prizadevamo pri snovanju cevovoda za čim manjši prenos vibracij. Običajno lahko to dosežemo s pravilnim načinom in mestom vpetja, s kombinacijo več cevi različnih materialov, z geometrijskimi lastnostmi cevovoda ter optimalnim potekom cevovoda.

0 INTRODUCTION

Nowadays, more and more cars have integrated air conditioners, where the individual parts are connected with a piping system. These piping systems are exposed to the vibrations of the compressor and the vibrations of the car body, caused by irregularities in the road surface. The vibrations of the piping system cause undesirable noise and the system may subsequently fail due to fatigue. In the designing of the piping system attention is focused on the minimization of the vibrations. This can be achieved with the proper types and positions of the supports, with a combination of pipes of different materials and the optimal shape of the pipe system in space.

Zaradi izredno širokega področja uporabe cevovodov, npr. kot izmenjevalniki toplote, pri hidravličnih inštalacijah, klimatskih napravah ipd. so se v zadnjih letih zahteve in želje po modeliranju prenosa vibracij po cevovodu zelo povečale. Pri pregledu literature zasledimo, da se večina avtorjev osredotoča na modeliranje bodisi ravne cevi ([1] in [2]), pri katerih poskušajo čim bolj natančno upoštevati interakcijo med tekočino in steno cevi, bodisi krožnega loka ([1], [3] in [4]), kjer pa fizikalni modeli še niso povsem dodelani. Zasledimo le nekaj virov, ki obravnavajo prenos vibracij po zapletenem prostorskem cevovodu ([5] in [6]). Avtorji običajno popišejo celoten cevovod z ravnimi končnimi elementi, pri katerih je upoštevan preprost model interakcije med tekočino in steno cevi.

V prispevku se bomo osredotočili na modeliranje prenosa vibracij po prostorskem cevovodu. Predpostavili smo, da lahko poljubno obliko cevovoda v prostoru popišemo s kombinacijo ravnih cevi in krožnih lokov. Uporabili bomo metodo prenosnih matrik (MPM), ki je v literaturi dobro poznana in velikokrat uporabljena metoda. Huang [7] je metodo uporabil za izračun kritične pretočne hitrosti, Walsh [8] je z MPM računal prenos moči po nosilcu in Koo [9] je uporabil MPM za določitev prenosa vibracij po cevovodu v prostoru, sestavljenem iz ravnih cevi.

Glavni namen prispevka je uporaba prenosnih matrik za modeliranje prenosa vibracij po prostorskem cevovodu. V ta namen so v drugem poglavju predstavljene gibalne enačbe in izpeljana prenosna matrika krožnega loka s tekočino. Nato je v tretjem poglavju podana primerjava rezultatov prenosa sile po cevovodu, dobljenih s prenosnimi matrikami in končnimi elementi za analizirani cevovod. Sledi poglavje eksperimentalnega ovrednotenja uporabljena modela krožnega loka. Kot zadnje sledi poglavje s sklepi.

1 METODA PRENOSNIH MATRIK

Zaradi celovitosti prispevka bomo najprej predstavili metodo prenosnih matrik. Metoda prenosnih matrik je uporabna za linijske sisteme, pri katerih nas zanima predvsem odziv sistema v končni točki *K* glede na harmonsko motnjo v začetni točki *Z*, primer tega je prikazan na sliki 1. Sistem najprej diskretiziramo na podelemente, katerih prenosne matrike **PM** poznamo. Celotno prenosno matriko sistema $\mathbf{PM}_S$ v nadaljevanju dobimo z verižnim množenjem posameznih prenosnih matrik, kakor to prikazuje enačba (1).

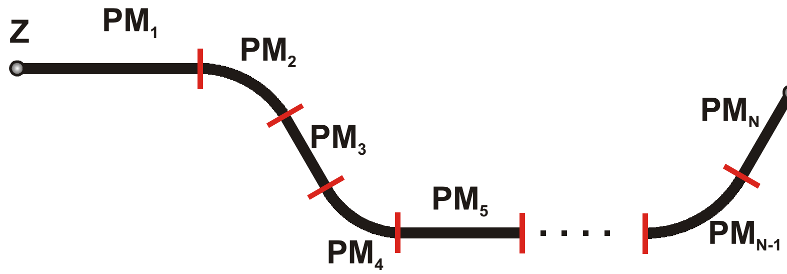
Vibration analyses of a piping system with a conveying fluid have received considerable attention in recent decades due to widespread applications in areas such as heat-exchanger tubes, hydraulic pipelines, air conditioners, etc. Extensive investigations have been carried out either on the subject of the vibrations of straight pipes conveying fluid ([1] and [2]), where the authors try to couple the motion of the pipe and the fluid accurately, or on curved pipes, conveying fluid ([1], [3] and [4]). Fewer studies have been made for the response analysis of a 3-dimensional complex piping system conveying fluid ([5] and [6]). The vibrations of the whole pipeline are usually studied by using conventional finite-element formulations.

This paper proposes a computational method to analyze the response of a 3-dimensional piping system. It has been assumed that the 3-dimensional piping system can be composed of a combination of straight and curved pipes. The transfer-matrix method (TMM) was applied to calculate the vibrations of the piping system. In [7] the method is used to calculate the critical flow velocity, the analysis of the power flow in beams is shown in [8], and in [9] the model for analyzing the vibrations of the straight pipe conveying the fluid is shown.

In Section 2 the governing equations are presented and the transfer matrix of the curved pipe including the fluid is derived. In Section 3 the comparison of the force transfer over the piping system is given using the derived transfer matrix and the finite-element method. The result for the curved pipe is verified by experiment. At the end the conclusions are presented.

1 TRANSFER-MATRIX METHOD

For the purposes of completeness this section gives a brief review of the transfer-matrix method. The TMM is mainly useful for one-dimensional systems, where only the response of the system at the end point K due to harmonic excitation at the start point Z is of interest, Fig 1. The system is initially discretized on the sub-elements, where the transfer matrices (**PM**) of those sub-elements are known. The global $\mathbf{PM}_S$ of the system can be further computed with the chain-product of the individual **PM**, Eq. 1.



Sl. 1. *Elementi cevne sestave in njihove prenosne matrike*
Fig. 1. *Elements of the piping system and their transfer matrices*

$$\mathbf{PM}_S = \mathbf{PM}_N \cdot \mathbf{PM}_{N-1} \cdots \mathbf{PM}_1 \quad (1).$$

Poznavanje prenosne matrike cevovoda $\mathbf{PM}_S$ omogoča določitev vektorja stanja na koncu cevovoda, točka *K*, če poznamo vektor stanja na začetku cevovoda, točka *Z*, kakor prikazuje enačba (2). Vektor stanja vsebuje vse pomike in elastične sile v izbranem krajišču:

Knowing the global transfer matrix of the piping system $\mathbf{PM}_S$ enables us to represent the piping system with pipe-end state vectors at points *Z* and *K*, Eq 2. In the pipe-end state vector all the elastic restoring forces and the displacements that influence the pipe-element end are introduced.

$$\mathbf{Z}_{SZ} = \mathbf{PM}_S \mathbf{Z}_{SK} \quad (2).$$

1.1 Prenosna matrika ravne cevi

Diferencialne enačbe, ki popisujejo nihanje ravne cevi s preprostim modelom tekočine, so podane v [1]. Enačbi, ki popisujeta prečni nihanji, sta dobljeni na temelju Euler-Bernoullijeve teorije upogiba nosilca, pri čemer je upoštevan tudi vpliv pretakajoče se tekočine. Tekočina je modelirana kot nestisljiva in neviskozna, kar je upravičeno v primerih obravnave dolgih cevovodov, pri katerih je razmerje dolžina/premer cevovoda veliko. Posledica takšnih predpostavk za tekočino je, da ne moremo modelirati tlačnih nihanj in dinamičnih učinkov tekočine v cevi. Za popis torzijskega nihanja je uporabljena Saint-Venantova teorija torzije, medtem ko je za osna nihanja uporabljena splošna teorija osnega nihanja nosilca, pri katerem upoštevamo tudi vpliv pretakajoče se tekočine. Na podlagi enačb, podanih v [1], je definirana prenosna matrika za ravno cev v [10].

1.1 Transfer matrix of a straight pipe

The governing differential equations for the vibrations of a uniform straight pipe with the plug-flow model of a fluid are given in [1]. The transverse vibrations are described with the Euler-Bernoulli beam theory, where the additional fluid effect is taken into account. The inviscid fluid-dynamics forces were derived according to the plug-flow approximation and the slender-body theory. The slender-body theory is valid for pipes with a large length-to-radius ratio. The drawback of such assumptions for the fluid is that the fluid effects cannot be modeled. The torsional vibrations are governed by the Saint-Venant theory. On the basis of the equations in [1], the transfer matrix for a straight pipe with a fluid was defined in [10].

1.2 Prenosna matrika krožnega loka

Gibalne enačbe, ki popisujejo nihanje krožnega loka s tekočino, prikazanega na sliki 2, so izpeljane v [10]. Enačbe so izpeljane ob predpostavki neupoštevanja učinka debelina-ukrivljenost oziroma upoštevamo linearni potek napetosti po prerezu, kar približno velja za krožne loke z razmerjem $R/D \geq 4$, kjer sta *D* premer cevovoda in *R* radij ukrivljenosti krožnega loka. Upoštevana pa je raztegljivost

1.2 Transfer matrix of a curved pipe

The governing equations for a curved pipe conveying a fluid, Figure 2, are given in [10]. The equations were governed by disregarding the thickness-curvature effect, which is valid for circular pipes with a ratio $R/D \geq 4$, where *D* is the diameter of the pipe and *R* is the radius of the curvature of the curved pipe. The extensibility conditions are taken into account. The inviscid fluid-dynamics forces were nevtralne črte. Uporabljen je enak model tekočine kakor pri ravni cevi, torej nestisljiva in neviskozna. Ker ima cev kolobarjasti prerez, ki je simetričen, dobimo le delno sklopljene gibalne enačbe.

Sklopljeni gibalni enačbi krožnega loka s tekočino, ki popisujeta nihanje znotraj ravnine ukrivljenosti, sta [10]:

derived according to the plug-flow approximation and the slender-body theory. Based on the Euler-Bernoulli hypothesis and because the cross-section is uniform and doubly symmetrical, the out-of-plane and the in-plane vibrations are uncoupled.

The governing differential equations for the in-plane vibrations are [10]:

$$\ddot{u}\left(m_p+m_f\right)+2m_f c\left(\dot{u}'+\frac{\dot{v}'}{R}\right)+m_f c^2\left(u''+2\frac{v'}{R}-\frac{u}{R}\right)+\frac{EI}{R}\left(v'''-\frac{u''}{R}\right)-EA_p\left(u''-\frac{v'}{R}\right)-A_f p_f\left(\frac{u}{R}-v'\right)\frac{1}{R}=q_s \quad (3)$$

$$\ddot{v}\left(m_p+m_f\right)+2cm_f\left(\dot{v}'-\frac{\dot{u}}{R}\right)+m_f c^2\left(v''-2\frac{u'}{R}-\frac{v}{R^2}\right)$$
$$+EI\left(v^{\mathrm{IV}}-\frac{u'''}{R}\right)+\frac{EA_p}{R}\left(u'+\frac{v}{R}\right)-A_f p_f\left(\frac{u'}{R}-v''\right)=\frac{1}{R}\left(c^2 m_f+A_f p_f\right)+q_r\ , \quad (4),$$

za nihanje zunaj ravnine ukrivljenosti [10]:

and for the out-of-plane vibrations are [10]:

$$\ddot{w}\left(m_p+m_f\right)+2m_f c\dot{w}'+EI\left(w^{\mathrm{IV}}-\frac{\phi''}{R}\right)-GI_0\left(\phi''+w''\right)+A_f p_f w''=q_z \quad (5)$$

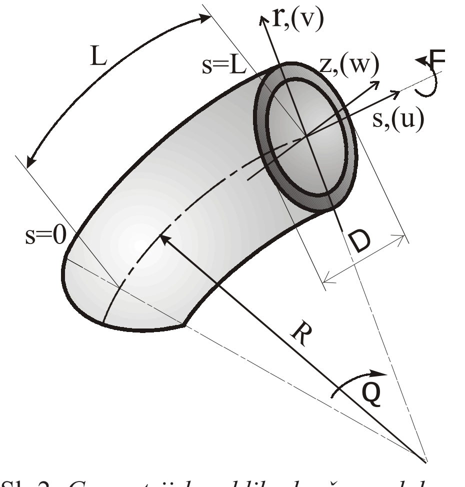
$$\rho_p I_0\ddot{\phi}+\frac{EI}{R}\left(\frac{\phi}{R}-w''\right)-GI_0\left(\phi''+\frac{w''}{R}\right)=q_t \quad (6),$$

kjer so: m_p, m_f dolžinski masi cevi in tekočine, c hitrost tekočine, A_p, A_f prečna prereza cevi in tekočine, u, v, w in ϕ so pomiki v osni smeri, v obeh prečnih smereh in zasuk okoli nevtralne osi, EA_p, EI in GI_0 so osna, prečna in torzijska togost, R polmer ukrivljenosti loka, p_f nadtlak tekočine, ρ_p gostota cevi in q_s, q_r, q_z zvezne obtežbe vzdolž cevi.

Nihanje krožnega loka bomo v nadaljevanju reševali ločeno, in sicer kot nihanje krožnega loka znotraj ravnine ukrivljenosti ter nihanje loka zunaj ravnine ukrivljenosti. Gibalni enačbi (3) in (4), ki popisujeta nihanje krožnega loka v ravnini sta sklopljeni parcialni diferencialni enačbi. V primeru,

where m_p and m_f are the masses per unit length for the pipe and the fluid, c is the fluid velocity, and A_p and A_f are the cross-sections of the pipe and the fluid. u, v and w denote the displacements of the pipe in the s, r and z directions and ϕ is the twist angle. The coefficients EA_p, EI in GI_0 represent the axial, the flexural and the torsional stiffnesses of the curved pipe, R is the radius of the curvature of the curved pipe, p_f is the internal pressure, ρ_p is the density of the pipe and q_s, q_r, q_z are the distributed excitation forces.

Because the in-plane and out-of-plane vibrations are uncoupled, the governing equations



Sl. 2. *Geometrijska oblika krožnega loka*
Fig. 2. *Geometry of the curved pipe*

da sta člena q_s in q_r enaka nič, je enačba (3) homogena, medtem ko je enačba (4) nehomogena. Nehomogeni člen v enačbi (4) predstavlja statično deformacijo krožnega loka, ki je posledica pretakajoče tekočine in nadtlaka v cevi. Izkaže se, da statična deformacija ne vpliva na prenos vibracij, zato lahko enačbo (4) nadalje obravnavamo kot homogeno.

Zanima nas le ustaljeno stanje nihanja loka glede na harmonično vzbujanje, zato iščemo le partikularni del rešitve gibalnih enačb (3) in (4). Za določitev partikularnih rešitev gibalnih enačb (3) in (4) izberemo nastavka, ki ju podajata enačbi (7) in (8):

are solved separately. The governing differential equations (3) and (4) for the in-plane vibrations are coupled and inhomogeneous. If there is no distributed load, Equation (3) becomes homogenous and Equation (4) inhomogeneous. The inhomogeneous part of Equation (4) is due to fluid effects and can be neglected, because only the steady-state vibrations are of interest.

Of most interest is the steady-state behavior, so consequently only the particular solution of Equations (3) and (4) is computed. For the harmonic excitation, the solutions for *u(s,t)* and *v(s,t)* of Equations (3) and (4) are written in the form:

$$u(s,t) = U(s)e^{j\omega t} \tag{7}$$

$$v(s,t) = V(s)e^{j\omega t} \tag{8},$$

kjer funkciji *U(s)* in *V(s)* podajata neznani obliki nihanja pri krožni frekvenci vzbujanja ω. Z vstavitvijo izrazov (7) in (8) v enačbi (3) in (4) in preurejanju dobimo naslednjo nesklopljeno diferencialno enačbo 6. reda v odvisnosti od spremenljivke *s*:

where *U(s)* and *V(s)* are the amplitudes of vibration and ω is the excitation frequency. After substituting Equations (7) and (8) into Equation (3) and (4), and after some mathematical manipulating of the equations, it is possible to obtain a sixth-order ordinary differential equation for *U(s)*:

$$K_6 U^{\mathrm{VI}} + K_5 U^{\mathrm{V}} + K_4 U^{\mathrm{IV}} + K_3 U''' + K_2 U'' + K_1 U' + K_0 U = 0 \tag{9},$$

kjer so koeficienti $K_6,...,K_0$ enaki:

where the coefficients $K_6,...,K_0$ are:

$$K_0 = A_p E\left[c^2 m_f + A_f p_f + \left(m_f + m_p\right) R^2\omega^2\right] - R^2\omega^2\left[c^2 m_f\left(m_p - m_f\right) + \left(m_f + m_p\right)\left(A_f p_f + \left(m_f + m_p\right) R^2\omega^2\right)\right] \tag{10}$$

$$K_1 = j m_f c R^2 \omega\left[2A_p E - c^2 m_f - A_f p_f + 3\left(m_f + m_p\right) R^2\omega^2\right] \tag{11}$$

$$\begin{aligned} K_2 = R^2&\left[-EI\left(m_f + m_p\right) + \left(c^2 m_f\left(3m_f + m_p\right) + A_f\left(m_f + m_p\right) p_f\right) R^2\right]\omega^2 \\ &+ A_p E\left[EI - R^2\left(-2c^2 m_f - 2A_f p_f + \left(m_f + m_p\right) R^2\omega^2\right)\right] \end{aligned} \tag{12}$$

$$K_3 = -jcm_f R^2\left[\left(c^2 m_f + A_f p_f\right) R^2 + E\left(I - 2AR^2\right)\right]\omega \tag{13}$$

$$K_4 = ER^2\left[A_p\left(2EI + \left(c^2 m_f + A_f p_f\right) R^2\right) + I\left(m_f + m_p\right) R^2\omega^2\right] \tag{14}$$

$$K_5 = jcEI\,\mathrm{m}_f\, R^4\omega \tag{15}$$

$$K_6 = A_p E^2 I R^4 \tag{16}.$$

Splošno rešitev enačbe (9) lahko zapišemo v naslednji obliki:

As is well known, the solution to equation (9) can also be written as:

$$U(s) = \sum_{i=1}^{6} D_i e^{\lambda_i s} \tag{17},$$

kjer so D_i neznani koeficienti, λ_i pa so koreni karakteristične enačbe (9). Ob znanem *U(s)*, pa lahko

where D_i are the unknown coefficients and λ_i are the roots of the characteristic equation (9). By knowing zapišemo, zaradi sklopljenosti enačb (3) in (4), še *V(s)* kot:

the *U(s)* due to the coupled governing equations (3) and (4), *V(s)* can also be obtained:

$$V(s)=\sum_{i=1}^{6}\Gamma_i D_i e^{\lambda_i s} \quad (18),$$

kjer so koeficienti Γ_i enaki:

where the coefficients Γ_i are:

$$\Gamma_i=\frac{jm_f cR\omega+\lambda_i\left(A_f p_f+c^2 m_f-EA_p\right)R+\lambda_i^3 EIR}{c^2 m_f+A_f p_f+\left(m_f+m_p\right)R^2\omega^2-jm_f cR^2\lambda_i\omega+E\left(I+A_p R^2\right)\lambda_i^2} \qquad i=1,\ldots,6 \quad (19).$$

Z uporabo enačb (17) in (18) zapišemo primarne spremenljivke na krajiščih krožnega loka v matrični obliki kot:

Using equations (17) and (18) the displacements of the pipe end can be written in matrix form:

$$\begin{Bmatrix}U_Z\\V_Z\\-V_Z'\\U_K\\V_K\\-V_K'\end{Bmatrix}=\begin{bmatrix}\alpha_{11}&\cdots&\alpha_{16}\\\vdots&\ddots&\vdots\\\alpha_{61}&\cdots&\alpha_{66}\end{bmatrix}\begin{Bmatrix}D_{V1}\\\vdots\\D_{V6}\end{Bmatrix} \quad (20),$$

kjer so:

where the coefficients a are:

$$\alpha_{1i}=\Gamma_i,\ \alpha_{2i}=1,\ \alpha_{3i}=-\lambda_i,\ \alpha_{4i}=\Gamma_i e^{\lambda_i L},\ \alpha_{5i}=e^{\lambda_i L},\ \alpha_{6i}=-\lambda_i e^{\lambda_i L}$$

Primarne spremenljivke so *U* osni pomik, *V* prečni pomik in *V'* zasuk. Indeksa *Z* in *K* primarnih spremenljivk pa ponazarjata začetno in končno krajišče. Enačbo (20) lahko v skrajšani obliki zapišemo kot:

The displacements are as follows: *U* is the axial displacement, *V* is the transversal displacement and *V'* is the slope. The letters Z and K indicate the start node and the end node, respectively. Equation (20) can be expressed further as:

$$\mathbf{W}_{in}=\mathbf{Q}_1\mathbf{D} \quad (21),$$

kjer je $\mathbf{W}_{in}$ vektor primarnih spremenljivk na krajiščih loka, **D** pa vektor neznanih koeficientov. Podobno kakor smo to zapisali v matrični enačbi (20) za primarne spremenljivke, zapišemo še sekundarne spremenljivke za krožni lok. Zveze med deformacijami in sekundarnimi spremenljivkami so podane v [8]. Matrična oblika je naslednja:

where $\mathbf{W}_{in}$ is the vector of the pipe-end displacements and **D** is the vector of the unknown coefficients. Using the Euler-Bernoulli theory for a curved pipe, the elastic restoring forces can be obtained with the equations given in [8]. Using these equations the elastic restoring forces at the end of the curved pipe are obtained with:

$$\begin{Bmatrix}-N_Z\\-M_Z\\-S_Z\\N_K\\M_K\\S_K\end{Bmatrix}=\begin{bmatrix}\beta_{11}&\cdots&\beta_{16}\\\vdots&\ddots&\vdots\\\beta_{61}&\cdots&\beta_{66}\end{bmatrix}\begin{Bmatrix}D_{V1}\\\vdots\\D_{V6}\end{Bmatrix} \quad (22),$$

kjer so:

where the terms β are:

$$\beta_{1i}=-A_p E\left(\frac{1}{R}+\lambda_i\Gamma_i\right),\ \beta_{21}=-EI\left(\frac{\lambda_i\Gamma_i}{R}-\lambda_i^2\right),\ \beta_{3i}=-EI\left(\frac{\lambda_i^2\Gamma_i}{R}-\lambda_i^3\right)$$

$$\beta_{4i}=A_p Ee^{\lambda_i L}\left(\frac{1}{R}+\lambda_i\Gamma_i\right),\ \beta_{5i}=EIe^{\lambda_i L}\left(\frac{\lambda_i\Gamma_i}{R}-\lambda_i^2\right),\ \beta_{6i}=EIe^{\lambda_i L}\left(\frac{\lambda_i^2\Gamma_i}{R}-\lambda_i^3\right).$$

Enačbo (22) zapišemo krajše v matrični obliki kot:

Equation (22) can be written as:

$$\mathbf{F}_{in}=\mathbf{Q}_2\mathbf{D} \quad (23),$$

kjer je $\mathbf{F}_{in}$ vektor neznanih sekundarnih spremenljivk z elementi: N osna sila, M upogibni moment in S prečna sila.

Razvidno je, da se vektor neznanih koeficientov $\mathbf{D}$ pojavlja tako v enačbi (21) kakor tudi v enačbi (23). Nepoznavanju vektorja $\mathbf{D}$ se lahko izognemo tako, da zapišemo neposredno povezavo med primarnimi in sekundarnimi spremenljivkami kot:

where $\mathbf{F}_{in}$ is the vector of the elastic restoring forces with the axial force N, the bending moment M and the shear force S.

The vector of the unknown coefficients appears in Equation (21) and also in Equation (23). To avoid the calculation of the unknown vector $\mathbf{D}$, equation (21) is substituted into equation (23) and the relation between the elastic restoring forces and the displacements is obtained as:

$$\mathbf{F}_{in} = \mathbf{Q}_2\mathbf{Q}_1^{-1}\mathbf{W}_{in} \qquad (24),$$

kjer $\mathbf{Q}_2\mathbf{Q}_1^{-1} = \mathbf{TM}_{in}$ pomeni togostno matriko velikosti 6×6, ki je funkcija frekvence. Togostno matriko $\mathbf{TM}_{in}$ je treba razširiti še za prostostne stopnje nihanja loka zunaj ravnine ukrivljenosti. Postopek določanja togostne matrike za nihanje loka izven ravnine je enak, kakor je prikazan za nihanje loka v ravnini. Tako dobimo togostno matriko krožnega loka $\mathbf{TM}^L$ velikosti 12×12, ki je definirana za krajevni koordinatni sistem krožnega loka in podaja povezavo med primarnimi in sekundarnimi spremenljivkami na začetnem in končnem krajišču, enačba (25):

where $\mathbf{Q}_2\mathbf{Q}_1^{-1} = \mathbf{TM}_{in}$ is the 6×6 dynamic stiffness matrix as a function of frequency. The dynamic stiffness matrix of the curved pipe has to be expanded for the out-of-plane vibration. The procedure for obtaining the dynamic stiffness matrix for the out-of-plane vibration is the same as was shown for the in-plane vibrations. Using this procedure, the final 12×12 dynamic stiffness matrix $\mathbf{TM}^L$ is obtained for the curved pipe. The relation between the displacement vector $\mathbf{W}$ and the vector of the elastic restoring forces $\mathbf{F}$ is given by the equation:

$$\mathbf{F} = \mathbf{TM}^L\mathbf{W} \qquad (25).$$

Metodologija prenosnih matrik vključuje v končni fazi verižno množenje prenosnih matrik posameznih elementov v rezultirajočo prenosno matriko $\mathbf{PM}_S$. Zaradi različne usmerjenosti posameznih elementov cevovoda je treba obravnavo prevesti na kinematične spremenljivke (primarne in sekundarne spremenljivke), definirane v absolutnem koordinatnem sistemu. Za spremembo vpeljemo transformacijsko matriko koordinatnih sistemov $\mathbf{Tc}$. Enačbo (25) lahko sedaj zapišemo v absolutnem koordinatnem sistemu v obliki:

With this dynamic stiffness matrix and the dynamic stiffness matrix for the straight pipe, the final dynamic stiffness matrix for the complex 3-dimensional piping system using the global assembly techniques can be constructed. To assemble the dynamic stiffness matrix of a pipe element in a local coordinate system into the global coordinate system the transformation matrix $\mathbf{Tc}$ is introduced, so Equation (25) can be transformed into the global coordinate system as:

$$\mathbf{Tc}^{\mathrm{T}} \cdot \mathbf{F} = \mathbf{Tc}^{\mathrm{T}} \cdot \mathbf{TM}^{\mathrm{L}} \cdot \mathbf{Tc} \cdot \mathbf{W} \qquad (26)$$

in v skrajšani obliki kot:

Or simply:

$$\mathbf{F}_G = \mathbf{TM}_G^L\mathbf{W}_G \qquad (27).$$

Indeks G ponazarja zapis enačbe v absolutnem koordinatnem sistemu. Na podlagi togostne matrike $\mathbf{TM}_G^L$ sedaj definiramo prenosno matriko $\mathbf{PM}_G^L$. Določimo jo s pomočjo delnega obrata dinamične togostne matrike tako, da v levem vektorju zberemo le spremenljivke, ki se pojavljajo na končnem krajišču in v desnem vektorju le spremenljivke, ki se pojavljajo v začetnem krajišču. Po delnem obratu dobimo:

The subscript G indicates that Equation (27) is defined in the global coordinate system. To obtain the transfer matrix from Equation (27) a partial inversion of the dynamic stiffness matrix is needed and further leads to the transfer matrix as:

$$\begin{Bmatrix} \mathbf{W}_{KG} \\ \mathbf{F}_{KG} \end{Bmatrix} = \begin{bmatrix} \mathbf{PM}_1 & \mathbf{PM}_2 \\ \mathbf{PM}_3 & \mathbf{PM}_4 \end{bmatrix} \begin{Bmatrix} \mathbf{W}_{ZG} \\ \mathbf{F}_{ZG} \end{Bmatrix} \qquad (28).$$

Enačbo (28) zapišemo kratko v obliki:

Equation (28) can be represented as:

$$\mathbf{Z}_K = \mathbf{PM}_G^L \mathbf{Z}_Z \tag{29}$$

kjer sta $\mathbf{Z}_K$ in $\mathbf{Z}_Z$ vektorja stanja na končnem in začetnem krajišču. $\mathbf{PM}_G^L$ je prenosna matrika krožnega loka.

V gibalnih enačbah krožnega loka dušenje ni vpeljano izrecno, ker upoštevamo le strukturno dušenje, preko vpeljanega kompleksnega modula elastičnosti.

where $\mathbf{Z}_K$ and $\mathbf{Z}_Z$ are the state vectors at the pipe-element ends.

The governing equations of the curved pipe are un-damped, because damping is modeled using the complex modulus of elasticity.

2 NUMERIČNI PREIZKUS

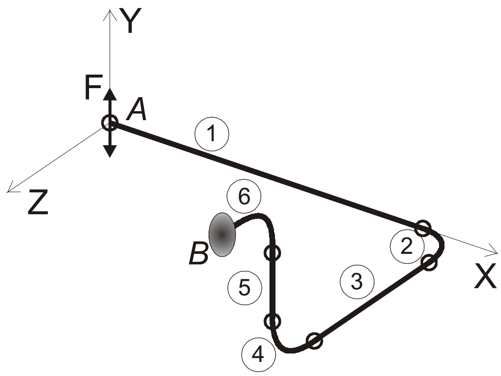
Na primeru cevovoda (sl. 3), smo s predstavljeno metodo določili prenosnost sile in jo primerjali s prenosnostjo, določeno z MKE, v programskem paketu ANSYS. Cevovod je sestavljen iz treh ravnih cevi (1, 3 in 5) ter treh krožnih lokov (2, 4 in 6). Podatki, ki smo jih uporabili pri analizi so naslednji: E=210 GPa, D=0,03 m, d=0,025 m, m_f=0,49 kg/m, m_p=1,695 kg/m. Dolžine ravnih cevi so: l_1=1 m, l_3=0,6 m in l_5=0,4 m in radiji lokov R_2=0,3 m, R_4=0,2 m in R_6=0,4 m. Uporabljeni programski paket ANSYS ne omogoča analize cevovoda s pretakajočo tekočino, zato je za analizirani primer vzeto, da tekočina miruje, c=0 m/s in ni pod nadtlakom. Z obravnavo po metodi prenosnih matrik smo celoten cevovod popisali s 6 elementi, medtem ko je v modelu KE cevovod modeliran s 60 ravnimi elementi *PIPE 16*. V obeh primerih smo analizirali prenos sile po cevi, iz točke *A* do točke *B*, v frekvenčnem področju od 10 do 250 Hz.

Prenos sile smo določevali z linearno prenosno funkcijo *TF*, ki je definirana kot $TF(\omega)=F_B(\omega)/F_A(\omega)$, kjer sta $F_A(\omega)$ amplituda vzbujevalne sile, $F_B(\omega)$ pa amplituda odzivne sile v

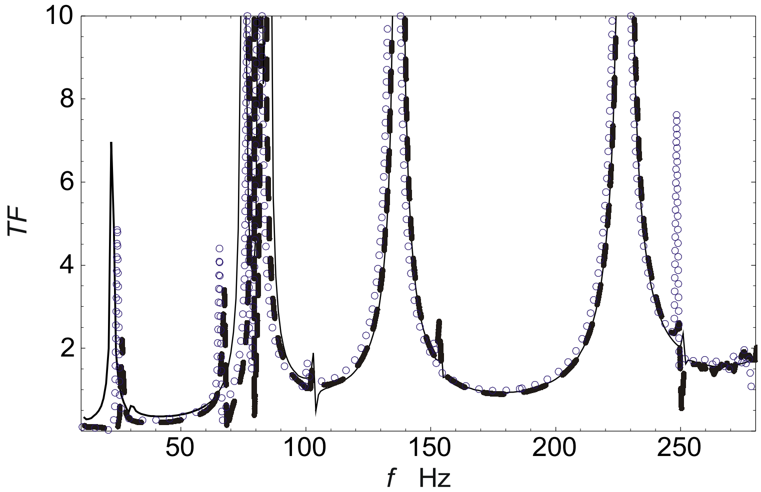
2 NUMERICAL EXPERIMENT

In this section the frequency-response analysis was carried out using the proposed transfer-matrix method and the FEM for the 3-dimensional piping system conveying a fluid, Fig. 3. The piping system was composed of three straight pipes (1,3 and 5) and three curved pipes (2, 4 and 6). The data used in the analyses were as follows: E=210 GPa, D=0.03 m, d=0.025 m, m_f=0.49 kg/m, m_p=1.695 kg/m. The lengths of the straight pipes were as follows: l_1=1 m, l_3=0.6 m and l_5=0.4, and the radii of curvature were R_2=0.3 m, R_4=0.2 m and R_6=0.4 m. The commercial package used for the FEM (Ansys) is not able to model the vibrations of pipes with a moving fluid. Consequently, only the case of a non-moving fluid was analyzed, c=0. The number of elements used in the transfer matrix approach was only 6, but with the finite-element method 60 straight pipe elements (PIPE 16) were used. The force transmissibility over the piping system, from point *A* to point *B*, was analyzed in both cases for the frequency range from 10 to 250 Hz.

For the computation of the force transmissibility, the transfer response function (*TF*) was defined as $TF(\omega)=F_B(\omega)/F_A(\omega)$, where $F_A(\omega)$ and $F_B(\omega)$ represent the forces at the points A and B



Sl. 3. *Primer 3D analiziranega cevovoda*
Fig. 3. *Analyzed numerical example*



Sl. 4. *Prenosna funkcija TF v smeri y; (a) MPM, c=0 (---), (b) MKE, c=0 (—) in (c) MPM, c=2,5m/s (ooo)*
Fig. 4. *Transfer function TF in the y direction: (a) MPM, c=0 (---), (b) MKE, c=0 (—) and (c) MPM, c=2.5m/s (ooo)*

točki *B* v odvisnosti od frekvence. Prenosna funkcija je prikazana na sliki 4. Iz grafa je razvidno, da prihaja do neujemanja vrednosti prenosnosti, kakor tudi lastnih frekvenc, med prenosnima funkcijama, določenima z MPM in MKE v nizkem frekvenčnem področju, medtem ko je ujemanje v višjem frekvenčnem področju boljše. Razhajanje prenosa sile, določene z MPM in MKE v nizkem frekvenčnem področju, je posledica sklopljenih nihanj krožnega loka. To nakazuje dodatna resonančna frekvenca v področju med 60 Hz in 90 Hz v primeru obravnave z MPM, ki se v primeru obravnave z MKE ne pojavi. Glede na to, da je prenosna matrika izpeljana na temelju točne rešitve gibalnih enačb, medtem ko pri končnih elementih je rešitev približna z izbranimi oblikovnimi funkcijami, sklepamo, da podaja metoda prenosnih matrik bolj natančne rešitve. S slike 4 je tudi razvidno, da se v primeru upoštevanja hitrosti pretoka tekočine v cevi *c*=2,5 m/s potek prenosne funkcije nekoliko spremeni, iz enakega vzroka pa se spremeni tudi lega resonančnih frekvenc.

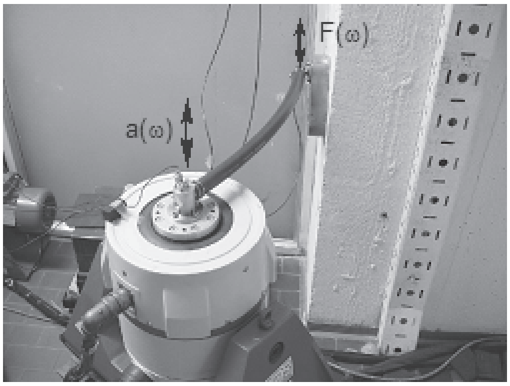
at the discrete frequencies. The transmissibility of the force obtained with the transfer-matrix method and the finite-element method is shown in Fig. 4. The finite-element method gives similar results to the transfer-matrix method in the higher-frequency regions but in the lower-frequency regions the difference is greater. The differences in the force transmissibility at low frequencies are believed to be due to the uncoupled axial and flexural vibration in the finite-element formulation. This shows in the additional resonance frequency that occurs from 60 Hz to 90 Hz in the transfer-matrix approach but not with the finite-element method. However, in the transfer-matrix approach the dynamic properties are derived exactly from the governing equations for the straight pipe and the curved pipe; therefore, the results from the transfer-matrix method are exact within the range of validity of the assumed governing equations and the plug-flow model. As can be seen from Fig. 4, example (c), the fluid effects can not be neglected.

3 PREIZKUS

Izbrani matematični model za popis vibracij krožnega loka cevi smo ovrednotili s preizkusom. V prvi fazi nas je zanimala pravilnost uporabljenega modela na primeru krožnega loka, v drugi fazi pa njegova primernost za napoved prenosa vibracij po krožnem loku. Pravilnost modela smo ovrednotili na kompozitni gumeni cevi znanih geometrijskih lastnosti. Za določitev prenosa prečnih vibracij in lastnih

3 EXPERIMENAL STUDY

The suitability and verification of the mathematical model used for the prediction of the vibration transmissibility over the curved pipes was tested with an experiment. The verification of the mathematical model was made on a curved rubber hose. The hose was excited with a sweep-sine signal in the frequency range from 15 to 300 Hz. At the excited end of the hose the amplitude



Sl. 5. *Preizkus za merjenje prenosnosti sile krožnega loka*
Fig. 5. *Experimental setup for measuring the vibration of the curved pipe*

frekvenc smo sistem vzbujali s sinusnim signalom v območju od 15 Hz do 300 Hz (prelet frekvenc). Pri meritvi smo zajemali amplitudo vzbujevalnega pospeška $a(\omega)$ na enem krajišču, na drugem pa preneseno amplitudo sile $F(\omega)$, kakor je prikazano na sliki 5.

Na temelju teh dveh merjenih spremenljivk smo definirali prenosno funkcijo kot:

of the acceleration $a(\omega)$ was measured and at the end the amplitude of the transferred force $F(\omega)$ was measured. The experimental set-up is shown in Fig 5.

The vibration transmissibility of the curved pipe is derived from the ratios of the measured excitations and responses as:

$$TF_M(\omega) = \frac{F(\omega)}{a(\omega)} \quad (30),$$

ki bo rabila za ugotavljanje ustreznosti uporabljenega matematičnega modela.

Za določitev prenosa vibracij preko loka z uporabo matematičnega modela je treba poznati poleg geometrijskih tudi snovne lastnosti gradiva. Geometrijski podatki so preprosto izmerljivi, medtem ko je določitev elastičnega modula E nekoliko večji problem. Obravnavani lok je namreč kompozit več plasti gume in ojačan z vlakni. Glede na prevladujoči delež gume v kompozitu le-ta izkazuje viskoelastične lastnosti. Lastnosti viskoelastičnega gradiva lahko v primeru harmonskih pomikov podajamo s kompleksnim elastičnim modulom, ki je funkcija frekvence. Imaginarni del kompleksnega modula elastičnosti predstavlja dušilne lastnosti materiala. Običajnega testa statičnega določanja elastičnega modula v tem primeru ne moremo uporabiti. Wei [11] navaja možnost ocenitve povprečnega elastičnega modula prereza na podlagi resonančne tehnike. Podal je zvezo za oceno modula elastičnosti, če poznamo izmerjene lastne frekvence ter geometrijske in snovne lastnosti gradiva:

Equation (30) will be used for the assessment of the suitability of the mathematical model.

For the calculation of the vibration transmissibility with the derived mathematical model, the proper geometrical and material characteristics need to be identified. Geometrical data can be easily obtained, while a measurement of the elastic modulus represents a slightly larger problem. The analyzed hose is made from different layers of rubber and is reinforced with a cord. This hose has viscoelastic properties because of the overall composition of the rubber. The viscoelastic properties can be described under harmonic load with a complex elastic modulus, which in many cases is frequency dependent. The imaginary part of the complex elastic modulus relates to the energy dissipation. The conventional static method for measuring the elastic modulus cannot be used in this case. Wei [11] proposed the resonance technique for determining the average elastic modulus for composite structures. The relation between the modulus of elasticity and the resonance frequencies is determined by means of the formula:

$$E_i = CWf_i^2 \quad (31),$$

kjer so: W masa preizkušanca, f_i^2 i-ta lastna frekvenca in C faktor oblike, definiran kot:

where W is the mass of the specimen, f_i^2 is the resonant frequency and C is the factor that can be written in the form:

$$C = \frac{4\pi^2 L^3}{n^4 I} \quad (32),$$

kjer so: L dolžina, n koeficient robnega pogoja in I vztrajnostni moment prereza. Elastični modul obravnavanega loka smo določili pri štirih lastnih frekvencah. Za popis elastičnega modula čez celotno obravnavano frekvenčno območje smo dobljene vrednosti interpolirali s kubičnim polinomom. Imaginarni del elastičnega modula smo ocenili po literaturi [12] in ga nato popravili glede na ujemanje prenosnih funkcij v okolici resonance. Ovrednotenje tako dobljenega elastičnega modula je bila izvedena na krožnem loku in ravni cevi različnih dolžin. Izmerjena in izračunana krivulja prenosnosti sile preko krožnega loka sta prikazani na sliki 6. Razvidno je, da je ujemanje med meritvami in rezultatom numeričnega modela zadovoljivo. Iz grafa je razvidno, da so amplitude funkcije prenosnosti določene z numeričnim modelom nekoliko manjše kakor pa to pokažejo meritve. Predvidevamo, da je razlika posledica izbranega poenostavljenega modela za določitev snovnih lastnosti cevovoda.

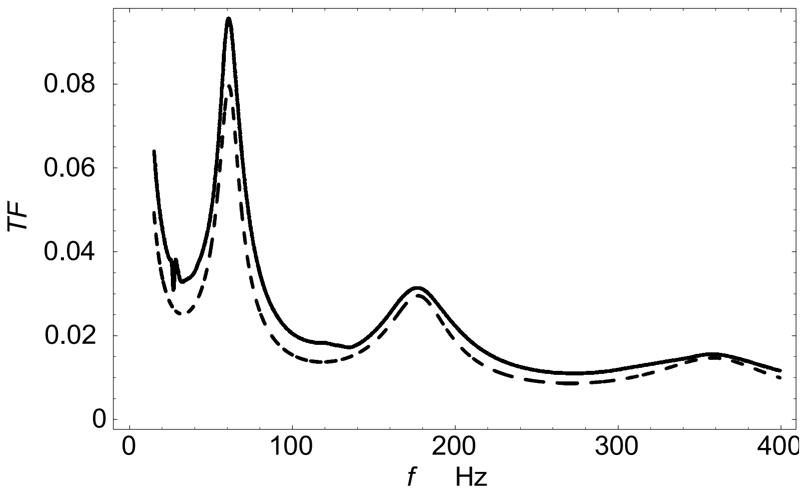
where L is the length of the specimen, n is a factor that depends on the boundary conditions, and I is the second area moment of the cross-section. The elastic modulus of the hose was estimated from the first four resonant frequencies. The values of the elastic modulus were interpolated above the entire frequency range of interest with a cubic polynomial. The imaginary part of the elastic modulus was assessed on the basis of the literature [12] and then corrected to fit the transmissibility function at the resonant region. The measured values of the elastic modulus were verified for the straight pipes of different lengths. The measured and calculated transfer functions are shown in Fig. 6. It is clear from Fig. 6 that the agreement between the transfer functions is good. The amplitudes of the transmissibility function obtained with the TMM are shown to be a little smaller than the amplitudes of the measured transfer function. We believe that the difference in the amplitudes of the transfer functions is due to the simplified model used for the material properties.

4 SKLEPI

V prispevku predstavljena metoda prenosnih matrik se izkaže za učinkovito metodo pri modeliranju prenosa vibracij preko poljubnega cevovoda v prostoru. Izpeljana prenosna matrika v drugem poglavju z uporabo prenosne matrike za ravno cev, definirane v [10], omogoča dodatno upoštevanje vpliva pretakajoče se tekočine in nadtlaka na prenos vibracij preko cevovoda.

4 CONCLUSIONS

The paper presents the use of the transfer-matrix method for modeling the vibration transmission over an arbitrary piping system in space. With the help of the derived matrix in Section 2 and the matrix for straight piping, which is shown in [10], we can consider the additional influence of the fluid flow and the fluid pressure on the vibration transmission over the piping system. The fluid model used in the analysis



Sl. 6. *Graf izmerjene in numerično dobljene prenosne funkcije: izmerjeno (—), MPM (---)*
Fig. 6. *Measured and calculated transfer functions: measurement (—) and TMM (---)*

Uporabljen model tekočine je preprost, zato omogoča le analizo vpliva tekočine na prenos vibracij, medtem ko dinamičnih učinkov tekočine s takšnim modelom ne moremo analizirati.

Iz analiziranega numeričnega primera vidimo, da dobimo po MPM podobne rezultate kakor po MKE, kar pomeni, da metoda s teoretičnega vidika deluje pravilno. Na podlagi analiziranega primera, pri katerem upoštevamo pretakajočo se tekočino, lahko sklepamo, da tega vpliva ne smemo zanemariti, predvsem za večje hitrosti tekočine.

Primerjava eksperimentalno in numerično določene prenosnosti potrdi pravilnost uporabljenega postopka, saj je ujemanje vrednosti prenosnosti kakor tudi vrednosti lastnih frekvenc precej dobro. Uporabljeni kompleksni elastični modul za popis snovnih lastnosti se izkaže v primeru harmonskega obremenjevanja viskoelastičnega gradiva za primernega.

Predstavljena metoda torej lahko uspešno rabi za iskanje optimalne sestave cevi različnih gradiv in geometrijske oblike, z namenom zmanjševanja prenosa vibracij po cevovodu. Omogoča tudi analizo že obstoječih cevovodov, pri katerih lahko spreminjamo režim delovanja (hitrost in tlak tekočine), z namenom zmanjševanja prenosa vibracij. V prihodnje bi bilo primerno preveriti prenos vibracij preko spojev dveh cevi, saj v obravnavanih numeričnih primerih predpostavljamo, da se vibracije prenašajo preko spojev brez dodatnih ojačitev ali izgub.

can only account for the influence of the fluid on the vibration transmission, while the additional observation of the dynamic effect of the fluid is not possible.

It is clear from Figure (4) that the results obtained with the TMM model and the FEM model give similar values regarding the vibration transmission function. From this we can conclude that the presented method gives the correct results from the theoretical point of view. The results obtained from the case study showed that the consideration of the fluid's influence on the piping system is very important, especially for high flow velocities.

The comparison of the experimental and numerical results for the vibration transmission verifies the correctness of the used approach, because the vibration transmissions function and the values of the natural frequencies give a satisfactory agreement. The use of the complex modulus of elasticity was found to be suitable for the purposes of accounting the viscoelastic material properties and the harmonic excitation of the piping system.

In general, the presented method proves to be very useful for the optimization of the shape of a piping system with different material properties and geometries, with the objective of minimizing the vibration transmission. The model can also be used for an analysis of an existing piping system, where the influence of the fluid velocity and the fluid pressure can be observed and varied for the purposes of minimizing the vibration transmission. In future work the joints of the piping substructures could be modeled and analyzed, because in the present paper the assumption of an ideal vibration transmission over the joint is made, which means there are no additional increases in the vibrations because of joints.

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5 LITERATURE

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